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# On the Formation Control of a Multi-Vehicle system

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*Abstract*—**In this paper, a novel approach to the formation control of multiple rectangular agents with limited communication ranges is presented. By utilizing an artificial potential function, the proposed control algorithm can guarantee fast formation performance and no collision among agents. Hence, the rectangular agents can collaborate to form a pre-defined shape of formation without collisions. A simulation result is conducted to demonstrate the effectiveness of the proposed algorithm.**

**Key words: Multi-agent systems; Robotics; Formation control; Multi-agent cooperative control.**

## I. Introduction

There has been a growing interest in formation control for multi-agent systems because of its potential applications in various fields such as target tracking [1], [2], environmental monitoring [3], [4], scalar field mapping [5], [6], intelligent transportation systems [7], [8], and task allocations [9], [10], etc. Each agent is required to follow a pre-defined trajectory to coordinate with other agents in order to create a specific formation. Various networks of the agents are combined from sensors, control algorithms and other dynamic factors which depend on specific purposes or application scenarios [6], [11].

Most existing work in the literature models the agents as single particle/point or elliptical/circular shapes. Circular approximation for long rectangular shapes may produce a conservative result for formation control of a multi-agent systems in the case where the agents have to travel through a narrow environment. For example, autonomous cars have to travel in the traffic lanes in the city in rush hours. This motivates us to consider rectangular agents where all four vertices associated with width and length of the rectangle are taken into account. This rectangular model can help remove redundant areas. The rectangular agent model can be beneficially applied to formation control of multiple autonomous cars in the future of intelligent transportation systems.

Typical multi-agent systems consist of three different agent shapes: circular, elliptical and rectangular, respectively. In the multi-agent system, formation control plays a critical role for pursuing cooperative tasks. Most existing work of multi-agent formation control has been focused on circular/disk agents [12]–[14] and elliptical agents [15], [16]. Additionally, extension to formation control for circular agents in noisy environments has been reported in [17], [18]. The recent review of formation control of multiple circular agents has been presented in [19]. All of the aforementioned formation control works mainly focus on circular and elliptical agents. A preliminary result on rectangular agents was reported in [20], in which the rectangular agent model was considered but a complete algorithm to avoid collisions was not addressed yet. Hence, in this paper, we investigate further results on the formation control of rectangular agents.

The main contribution of this paper is to provide a theoretical and computational framework for design and analysis of distributed formation control algorithm for multiple rectangular agents. A new approach to a rectangular agent model is proposed with a consideration of all four corners of the agent to avoid redundant areas. The proposed distributed formation control algorithm can allow rectangular agents to form different formation shapes such as line, lattice or circular shapes, respectively. The convergence analysis for the proposed work is given.

## II. Multi-Rectangular Agent Model

Consider two rectangles $i$ and $j$, where $(x_i, y_i)$ denotes the position of the center $O_i$ and $\phi_i$ is the heading angle of rectangle $i$ as shown in Figure 1. The length and width of rectangle $i$ are $b_i$ and $a_i$, respectively. Furthermore, $O_iX_iY_i$ denotes a coordinate frame associated with agent $i$. $(x^i_A, y^i_A)$ is the coordinate of point $A$ in the frame $O_iX_iY_i$. The notation for rectangle $j$ is similar. For brevity, $c$ and $s$ stand for $\cos$ and $\sin$, respectively.

The coordinates of four vertices of rectangle $i$ are

$$(x_{A_{i1}}, y_{A_{i1}}) = (x_i + \frac{b_i}{2}c\phi_i - \frac{a_i}{2}s\phi_i, y_i + \frac{b_i}{2}s\phi_i + \frac{a_i}{2}c\phi_i) \quad (1)$$

$$(x_{A_{i2}}, y_{A_{i2}}) = (x_i + \frac{b_i}{2}c\phi_i + \frac{a_i}{2}s\phi_i, y_i + \frac{b_i}{2}s\phi_i - \frac{a_i}{2}c\phi_i) \quad (2)$$

$$(x_{A_{i3}}, y_{A_{i3}}) = (x_i - \frac{b_i}{2}c\phi_i + \frac{a_i}{2}s\phi_i, y_i - \frac{b_i}{2}s\phi_i - \frac{a_i}{2}c\phi_i) \quad (3)$$

$$(x_{A_{i4}}, y_{A_{i4}}) = (x_i - \frac{b_i}{2}c\phi_i - \frac{a_i}{2}s\phi_i, y_i - \frac{b_i}{2}s\phi_i + \frac{a_i}{2}c\phi_i) \quad (4)$$

Let $\phi_{ji} = \phi_j - \phi_i$ , $x_{ji} = x_j - x_i$, and $y_{ji} = y_j - y_i$. The distance $d_{ij}$ between the two rectangles $i$ and $j$ is defined by the smallest distance from one vertex of one rectangle to the other rectangle. So, in the following, the distance from one vertex to one agent is derived.

Denote

$$p_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}, \quad (5)$$

$$p_{ij} = p_i - p_j. \quad (6)$$

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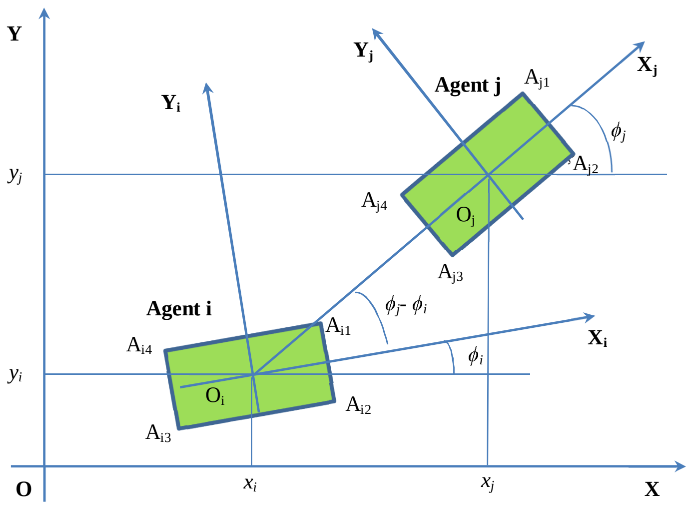

Fig. 1: Two rectangular agents with their coordinates in the frame $OXY$ (adopted from [20]).

Similarly,

$$p_{A_{ik}} = \begin{bmatrix} x_{A_{ik}} \\ y_{A_{ik}} \end{bmatrix} \tag{7}$$

for $k = 1, ..., 4$. The coordinate of a vertex $A_{jk}$ in the frame $O_i X_i Y_i$ for $k = 1, ..., 4$ is

$$p^i_{A_{jk}} = R(\phi_i)(p_{A_{jk}} - p_i) \tag{8}$$

where

$$R(.) = \begin{bmatrix} c(.) & s(.) \\ -s(.) & c(.) \end{bmatrix}, \tag{9}$$

and

$$p^i_{A_{jk}} = \begin{bmatrix} x^i_{A_{jk}} \\ y^i_{A_{jk}} \end{bmatrix}. \tag{10}$$

Let

$$f_{xi}(p_{A_{jk}}) = \begin{cases} |x^i_{A_{jk}}| - \frac{b_i}{2} & \text{if } |x^i_{A_{jk}}| > \frac{b_i}{2} \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

and

$$f_{yi}(p_{A_{jk}}) = \begin{cases} |y^i_{A_{jk}}| - \frac{a_i}{2} & \text{if } |y^i_{A_{jk}}| > \frac{a_i}{2} \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

The distance from $A_{jk}$ to rectangle $i$ is given as

$$\xi_i(p_{A_{jk}}) = \sqrt{f_{xi}(p_{A_{jk}})^2 + f_{yi}(p_{A_{jk}})^2}. \tag{13}$$

Hence, the distance between two rectangular agents $i$ and $j$ is defined as

$$d_{ij} = \min(\min_k(\xi_i(p_{A_{jk}})), \min_k(\xi_j(p_{A_{ik}}))) \text{ for } k = 1, ..., 4. \tag{14}$$

It can be seen that $d_{ij}$ is a nonsmooth function of $p_{ij}$, $\phi_{ij}$, and $\phi_i$.

## III. Problem Statement

### A. System model

For the sake of simplicity, we assume that each agent has the following dynamics:

$$\dot{q}_i = u_i, \tag{15}$$

for all $i \in N$, where $N$ is the set of all agents in the group. Denote $u_i = [u_{xi}, u_{yi}, u_{\phi i}]^T$ as the control input and $q_i = [x_i, y_i, \phi_i]^T$ as the position and orientation of agent $i$.

Each agent in the network is modeled as follows:
**(a)** The formation trajectory of agent $i$ is

$$q_{if}(s_{if}) = [x_{if}(s_{if}), y_{if}(s_{if}), \phi_{if}(s_{if})]^T \tag{16}$$

where $s_{if}$ is a parameter.

Agent $i$ is guided by a reference trajectory to coordinate with other agents to form a pre-defined formation. The reference trajectory of agent $i$ is denoted as

$$q_{id}(t) = [x_{id}(t), y_{id}(t), \phi_{id}(t)]^T. \tag{17}$$

The distance $d_{ijf}$ between trajectories of agents $i$ and $j$ satisfies the following condition:

$$d_{ijf} \geq \delta_{ijf}, \tag{18}$$

where $\delta_{ijf}$ is a positive constant. Note that $q_{id}(t)$ is designed to guide agent $i$ to track $q_{if}(t)$.

**(b)** Agents $i$ and $j$ possess circular communication areas whose centers are at $O_i$ and $O_j$ with radii $R_i$ and $R_j$. They exchange information on $q_i$ and $q_{id}$ in their communication ranges, whose radii satisfy the following condition:

$$\min(R_i, R_j) \geq \delta_{ijR} + \frac{\sqrt{a_i^2 + b_i^2}}{2} + \frac{\sqrt{a_j^2 + b_j^2}}{2}, \tag{19}$$

where $\delta_{ijR}$ is a positive constant for all $(i, j) \in N$ and $j \neq i$.

**(d)** At the initial time $t_0 \geq 0$, all the agents are kept away far enough from each other, i.e.

$$d_{ij}(t_0) \geq \delta_{ij0}, \tag{20}$$

where $\delta_{ij0}$ is a positive constant, and $d_{ij}(t_0)$ is the distance between two agents $i$ and $j$, which is derived in the previous section, evaluated at $t = t_0$.

### B. Control Objective

Our objective is to design a distributed control law for each agent for the formation control problem. Hence, for each agent $i$, a control law $u_i$ is constructed based on its positions and angles as well as its neighbor agents such that $q_i$ tracks $q_{if}$ while avoiding collisions with the neighbors. Specifically, $u_i$ is designed such that

$$\lim_{t \to \infty} (q_i(t) - q_{if}(t))(t) = 0, \quad d_{ij}(t) \geq \delta_{ij} \tag{21}$$

for all $(i, j) \in N$, $i \neq j$, and $t \geq t_0 \geq 0$, where $\delta_{ij}$ is a positive constant.

## IV. Formation Control Design

### A. Potential function

Similar to [16], we propose a potential function, which will then be employed to design a control law for each agent. A potential function is composed of a goal function $\gamma$ and a collision avoidance function $\beta$

$$\varphi = \gamma + \beta \tag{22}$$

The goal function is designed as follows

$$\gamma = \sum_{i=1}^{N} \gamma_i \tag{23}$$

where

$$\gamma_i = \frac{k_1}{2}((x_i - x_{id})^2 + (y_i - y_{id})^2) + \frac{k_2}{2}(\phi_i - \phi_{id})^2 \quad (24)$$

with positive constants $k_1, k_2$.

The collision avoidance function is designed as

$$\beta = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \beta_{ij} \quad (25)$$

where $\beta_{ij}$ is a function of $d_{ij}$, and it is designed as

$$\beta_{ij} = k_{ij} \frac{1 - h_{ij}(\Delta_{ij}, a_{ij}, b_{ij})}{\Delta_{ij}} \quad (26)$$

where $k_{ij}$ is a positive parameter, $\Delta_{ij} = d_{ij}^2$ and $h_{ij}(\Delta_{ij}, a_{ij}, b_{ij})$ is a $p$-times differentiable smooth step function. A choice of $h_{ij}$ can be taken as in [16]. The constant $a_{ij}$ and $b_{ij}$ are chosen such that

$$0 < a_{ij} < b_{ij} \leq \min(\delta_{ijd}, \delta_{ijR}) - \mu_{ij} = \tau_{ijd}, \quad (27)$$

where $\delta_{ijd}$, $\delta_{ijR}$ are given in (18) and (19), and $\mu_{ij}$ is a positive constant.

### B. Control design

Let us define $p_{id} = [x_{id}, y_{id}]^T$, and $p_{ijd} = p_{id} - p_{jd}$ for all $(i, j) \in N$ and $j \neq i$. Substituting (24) and (26) into (22), we obtain

$$\begin{aligned} \varphi \;&=\; \frac{k_1}{2} \sum_{i=1}^{N} (p_i - p_{id})^2 + \frac{k_2}{2} \sum_{i=1}^{N} k_2 (\phi_i - \phi_{id})^2 \\ &\quad + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \beta_{ij}. \end{aligned} \quad (28)$$

Differentiating both sides of (28), we obtain

$$\begin{aligned} \dot{\varphi} \;&=\; \sum_{i=1}^{N} \Big[k_1(p_i - p_{id}) + D_i^\beta\Big]^T (\dot{p}_i - \dot{p}_{id}) \\ &\quad + \sum_{i=1}^{N} \Big[k_2(\phi_i - \phi_{id}) + E_i^\beta\Big] (\dot{\phi}_i - \dot{\phi}_{id}) \\ &\quad + D_i^\beta \dot{p}_{id} + E_i^\beta \dot{\phi}_{id}, \end{aligned} \quad (29)$$

where

$$D_i^\beta \;=\; -\sum_{j=1}^{i-1} \beta'_{ji} G_{ji}^\beta + \sum_{j=i+1}^{N} \beta'_{ij} G_{ij}^\beta \quad (30)$$

$$E_i^\beta \;=\; -\sum_{j=1}^{i-1} \beta'_{ji} H_{ji}^\beta + \sum_{j=i+1}^{N} (\beta'_{ij} (H_{ij}^\beta + L_{ij}^\beta)), \quad (31)$$

$$(32)$$

$$G_{ij}^\beta \;=\; \frac{\partial \Delta_{ij}}{\partial p_{ij}} (R(\phi_{id}) p_{ij}, \phi_{ij}, \phi_i - \phi_{id}), \quad (33)$$

$$H_{ij}^\beta \;=\; \frac{\partial \Delta_{ij}}{\partial \phi_{ij}} (R(\phi_{id}) p_{ij}, \phi_{ij}, \phi_i - \phi_{id}), \quad (34)$$

$$L_{ij}^\beta \;=\; \frac{\partial \Delta_{ij}}{\partial \phi_i} (R(\phi_{id}) p_{ij}, \phi_{ij}, \phi_i - \phi_{id}), \quad (35)$$

$$(36)$$

and

$$\beta'_{ij} = \frac{\partial \beta_{ij}}{\partial \Delta_{ij}}. \quad (37)$$

Let

$$\begin{aligned} \Omega_{pi} \;&=\; k_1(p_i - p_{id}) + D_i^\beta, \\ \Omega_{\phi i} \;&=\; k_2(\phi_i - \phi_{id}) + E_i^\beta. \end{aligned} \quad (38)$$

From (29), we choose a control law for agent $i$ as follows

$$\begin{aligned} \begin{bmatrix} u_{xi} \\ u_{yi} \end{bmatrix} \;&=\; \rho_i(-c_1 \Omega_{pi} + \dot{p}_{id}), \\ u_{\phi i} \;&=\; \rho_i(-c_2 \Omega_{\phi i} + \dot{\phi}_{id}), \end{aligned} \quad (39)$$

where $c_1$ and $c_2$ are positive constants, $\rho_i$ is a parameter determined later, which is 0 or 1. When collision avoidance is active, we can choose a reference trajectory for agent $i$ such that $\dot{q}_{id} = 0$ and $\rho_i = 0$. As a result,

$$D_i^\beta \dot{p}_{id} + E_i^\beta \dot{\phi}_{id} = 0. \quad (40)$$

We denote such a reference trajectory as $q_{id}$, which will be designed later. When no collision/obstacle avoidance is active, $q_{id} = q_{if}$ where $q_{if}$ is the formation reference of agent $i$.

### C. Convergence Analysis

The guarantee of collision avoidance among rectangular agents and the convergence of formation performance are presented in the following theorem.

*Theorem 1:* Under System Model III-A, the closed-loop system under the control input vector given in (39) for agent $i$ satisfies the following results:

1) There are no collisions between any agents and obstacle avoidance is guaranteed. In addition, the closed-loop system is forward complete.
2) The velocity of each agent satisfies

$$\begin{aligned} &\lim_{t \to \infty} (\dot{x}_i - \dot{x}_{id}) = 0 \\ &\lim_{t \to \infty} (\dot{y}_i - \dot{y}_{id}) = 0 \\ &\lim_{t \to \infty} (\dot{\phi}_i - \dot{\phi}_{id}) = 0. \end{aligned} \quad (41)$$

***Proof:*** Under the control law (39), the closed-loop system of (15) is given as

$$\begin{aligned} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} \;&\in\; \rho_i(-c_1 \Omega_{pi} + \dot{p}_{id}), \\ \dot{\phi}_i \;&\in\; \rho_i(-c_2 \Omega_{\phi i} + \dot{\phi}_{id}), \end{aligned} \quad (42)$$

and the derivative of $\dot{\varphi}$ is given as

$$\dot{\varphi} \in -k_1 \sum_{i=1}^{N} \rho_i \Omega_{pi}^T \Omega_{pi} - k_2 \sum_{i=1}^{N} \rho_i \Omega_{\phi i}^T \Omega_{\phi i}. \quad (43)$$

Equation (43) shows that $\dot{\varphi} \leq 0$. Hence, according to the extended LaSalle's Invariance Principle for switched nonlinear systems [21], $\varphi$ is bounded for $t \geq t_0 \geq 0$ and the state variables converge asymptotically to the set where $\dot{\varphi} = 0$. Since $\varphi$ is bounded, $\beta_{ij}$ is bounded. This implies $d_{ij} > 0$. Hence, no collision takes place between any two agents and obstacle avoidance is satisfied. Furthermore, the boundedness of $\varphi$ for $t \geq t_0 \geq 0$ implies the boundedness of $(p_i - p_{id})$ and $(\phi_i - \phi_{id})$.

In other words, or $(q_i - q_{id})$ is bounded for $t \geq t_0 \geq 0$. Hence, the closed-loop system is forward complete.

On the other hand, $\dot{\varphi} = 0$ implies $\Omega_{pi} = 0$ and $\Omega_{\phi i} = 0$. Using these values in (42), we obtain the expressions in (41). ■

### D. Reference trajectory design for collision avoidance

When collision avoidance is active, a reference trajectory $q_{id}$ is defined for agent $i$ to pass possible collisions. During this phase, $\dot{q}_{id} = 0$. For each rectangular agent, there are 4 escape references associated with 4 vertexes, for which another agent with higher priority can follow.

Denote $V^i_{A_{jk}}$ be an escape reference associated with $A_{jk}$ of agent $j$ for agent $i$. The coordinates of $V^i_{A_{jk}}$ are given as follows

$$p^T_{V^i_{A_{j1}}} = (x_j + (\frac{b_j}{2} + \alpha_i)c\phi_j - (\frac{a_j}{2} + \alpha_i)s\phi_j, y_j + (\frac{b_j}{2} + \alpha_i)s\phi_j + (\frac{a_j}{2} + \alpha_i)c\phi_j) \quad (44)$$

$$p^T_{V^i_{A_{j2}}} = (x_j + (\frac{b_j}{2} + \alpha_i)c\phi_j + (\frac{a_j}{2} + \alpha_i)s\phi_j, y_j + (\frac{b_j}{2} + \alpha_i)s\phi_j - (\frac{a_j}{2} + \alpha_i)c\phi_j) \quad (45)$$

$$p^T_{V^i_{A_{j3}}} = (x_j - (\frac{b_j}{2} + \alpha_i)c\phi_j + (\frac{a_j}{2} + \alpha_i)s\phi_j, y_j - (\frac{b_j}{2} + \alpha_i)s\phi_j - (\frac{a_j}{2} + \alpha_i)c\phi_j) \quad (46)$$

$$p^T_{V^i_{A_{j4}}} = (x_j - (\frac{b_j}{2} + \alpha_i)c\phi_j - (\frac{a_j}{2} + \alpha_i)s\phi_j, y_j - (\frac{b_j}{2} + \alpha_i)s\phi_j + (\frac{a_j}{2} + \alpha_i)c\phi_j) \quad (47)$$

where

$$\alpha_i = \frac{\sqrt{b_i^2 + a_i^2}}{2} + \eta_i \quad (48)$$

with $\eta_i > 0$ being a safety parameter. The heading angle reference $\phi_{id} = \phi_{if}(t_{ij})$ where $t_{ij}$ is the time when the collision avoidance is activated.

Denote $\sigma_i$ as the orientation of agent $i$. This variable is employed to determine which direction agent $i$ should follow to avoid possible collisions with other agents. Initially, $\sigma_i = 0$. If the agent $i$ moves counterclockwise, $\sigma_i = 1$. If it moves clockwise, $\sigma_i = -1$. Let $I_{ij}$ be an engagement index of agent $i$ and agent $j$ when collision avoidance is active. When $I_{ij} = 1$, one agent with less priority will stand still and act as an obstacle.

Let $d(O_i, O_j)$ be the distance between two objects $i$ and $j$. Let $q_{if}$ be the formation reference for agent $i$. Our purpose is to design $q_{id}$ such that agent $i$ avoid any possible collision and eventually track its formation reference.

In the beginning, set $\sigma_i = 0$, $I_{ij} = 0$ for $i = 1, ..., N$, and $j = 2, ...N$ $(i \neq j)$.

We have the following algorithm.

**for** each iteration $t = t_l$ **do**
**for** each agent $i$ **do**
**for** each agent $j$ $(i \neq j)$ **do**
**if** $d(O_i, O_j) < \mu_{ij}$ where $\mu_{ij} \geq b_{ij}$ is a safety distance **then**
**if** $I_{ij} = 0$ **then**
record the time $t_{ij}$
**end**
set $I_{ij} = 1$
**end**
**end**
Let $t_e = \max t_{ij}$. Define a coordinate transformation $p^e = R(\phi_e)(p - p_i(t_e))$ where $\phi_e = atan(\frac{y_{id}(t_e) - y_i(t_e)}{x_{id}(t_e) - x_i(t_e)})$.
Let $P(i) \in \mathbb{N}$ be a priority function of agent $i$, which satisfies $P(i) \neq P(j)$ if $i \neq j$.
**for** each agent $j$ $(i \neq j)$ **do**
**if** $I_{ij} = 1$ **then**
**if** $P(i) < P(j)$ **then**
$\rho_i = 0$
**else**
$\rho_i = 1$
**end**
**if** $\sigma_i = 0$ **then**
calculate $\Lambda = \max_v x^e_{A_{jv}} + \min x^e_{A_{jv}}$
**if** $\Lambda \geq 0$ **then**
set $\sigma_i = -1$
**else**
set $\sigma_i = 1$
**end**
**end**
**end**
**end**
**if** $\sigma_i \neq 0$ **then**
**if** $|q_{id} - q_i(t_l)| < \epsilon$ where $\epsilon$ is a small positive number **then**
set $t_e = t_l$
**end**
**for** each agent $j$ $(i \neq j)$ **do**
**if** agent $i$ has higher priority than agent $j$
**if** distance from agent $j$ to the line connecting $O_i$ and $O_{fi}$ is larger than $\mu_{ij}$ and $I_{ij} = 1$ where $O_{fi}$ is the reference point whose coordinate is $q_{if}$ **then**
$I_{ij} = 0$ and $\sigma_i = 0$.
**end**
**elseif** $d_{ij} > \mu_{ij}$ and $I_{ij} = 1$ **then**
$I_{ij} = 0$ and $\sigma_i = 0$.
**end**
**end**
**end**
**if** $\sum I_{ij} = 0$ **then**
set $\sigma_i = 0$, $q_{id} = q_{if}$.
**else**
**for** each agent $j$ $(i \neq j)$ **do**
Find $\min_{j,v}(|x^e_{V^i_{A_{jv}}}|, |x^e_{V^i_{A_{objv}}}|)$ subject to $I_{ij} = 1, \sigma_i x^e_{V^i_{A_{jv}}} > 0$.
Let the solution be $V^i_{A_m in}$.
Set $p_{id} = p_{V^i_{A_{min}}}$.
Set $\phi_{id} = \phi_{if}(t_e)$.
**end**
**end**

## V. Simulation results

In this section we test our proposed formation control algorithm for free space where we investigate a typical case of a lattice formation. The proposed algorithm can work for many other formation shapes such as V-shape and $\infty$ shape, however due to the similarity and limited space we do not report here.

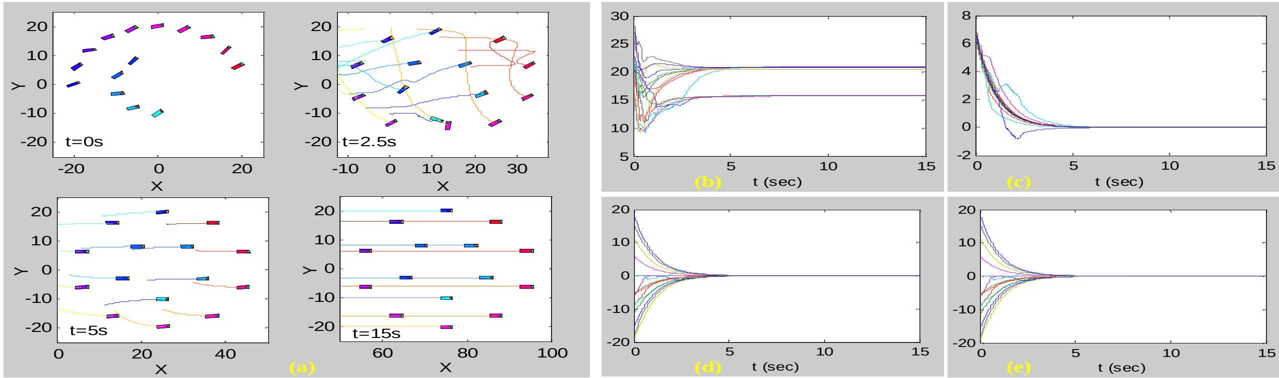

Fig. 2: (a) The snapshots of agent's movements and the agreement of agent's positions and orientations; (b) Distance representative $d_{ij}^*$; (c) Tracking errors in the heading angles; (d) Tracking errors in the x coordinate; (e) Tracking errors in the y coordinate.

In this example, we initially deploy 15 rectangular agents randomly with $a_i = 1$, $b_i = 3$. The parameters of the algorithm are chosen as: $\eta = 0.001$, $\epsilon = 0.0001$, $\mu_{ij} = 0.7$. An agent whose number is smaller than that of another agent has higher priority. The initial conditions are $q_i(0) = [R_0 \sin(\frac{2\pi(i-1)}{R_0} + \pi), R_0 \cos(\frac{2\pi(i-1)}{R_0} + \pi), 2\pi + \text{rand}(.)]^T$ with $R_0 = 10$ for $i = 1, \ldots, 5$ and $R_0 = 20$ for the other agents, and rand(.) a random number between 0 and 1. The control parameters are chosen as $C_1 = 10^4 \text{diag}(1,1)$, $K_1 = C_1^{-1}$, $c_2 = 10^4$, $k_2 = 1/c_2$. We implement a lattice formation by choosing $q_{if} = [x_{if}, R_0 \cos(\frac{4\pi(i-1)}{R_0} + \pi), 0]^T$ with $R_0 = 10$ for $i = 1, \ldots, 5$ and $R_0 = 20$ for the other agents. We choose $x_{if} = s_{if} - R_0 \sin(\frac{4\pi(i-1)}{R_0} + \pi)$. The signal $s_{if}(t)$ is given as:

$$\dot{s}_{if}(t) = -50(s_{if}(t) - s_{of}(t)) + \dot{s}_{of}(t), \tag{49}$$

where $s_{of} = 5t$ and $\dot{s}_{of} = 5$. The parameters of $\beta_{ij}$ in (26) are taken as $a_{ij} = 0$ and $b_{ij} = 0.3$. The parameter $k_{ij}$ is $10^{-5}$. Denote

$$d_{ij}^* = (\prod_{j=1, j\neq i}^{N} d_{i,j})^{1/(N-1)} \tag{50}$$

as the distance representative of agent $i$. Figure 2 (a) shows 12 rectangular agents forming a lattice formation. We can see that they can form a completed lattice shape after about 7 seconds. This clearly demonstrates the efficiency of the proposed control algorithm. Figure 2 (b) shows that the distance $d_{ij}^*$ (defined above) representatives of the rectangular agents converge to an equilibrium. The state errors of the agents, i.e. $(x_i - x_{id})$, $(y_i - y_{id})$, $(\phi_i - \phi_{id})$, are illustrated in Figure 2 (c, d, e).

## VI. Conclusions

A distributed control algorithm for multiple rectangular agents with limited communication ranges has been presented. A predefined collision-free formation can be obtained using the proposed control law. The convergence analysis of the proposed control algorithm is provided. A simulation result for the case of a lattice formation in free space has been conducted to illustrate the efficiency of the proposed control algorithm.

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