Automatica 100 (2019) 75–81

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Dynamic task allocation in multi-robot coordination for moving target tracking: A distributed approach ☆

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ARTICLE INFO

Article history:
Received 4 November 2016
Received in revised form 2 June 2018
Accepted 18 October 2018
Available online 22 November 2018

Keywords:
Dynamic task allocation
k-winners-take-all (k-WTA)
Distributed control
Target tracking
Nonlinear filters

ABSTRACT

A new coordination control is developed in this paper for multiple non-holonomic robots in a competitive manner for target tracking with limited communications. In this proposed control approach, only winners of the competition are allocated the task and activated to move towards the target. A distributed coordination model is proposed and its stability is proved in theory. Inspired by the besieging behaviors in social animals for predating, an effective strategy to handle the situation with higher target speed than trackers is also proposed and verified to be extraordinarily effective.

1. Introduction

As two aspects of the coordination behavior of a group, competition and cooperation of living beings, such as swarms of bees, flocks of birds, or even the troops in military, have certain advantages, including avoiding predators, increasing the chance of finding food, saving energy, besieging and capturing preys. For example, for the task of capturing a prey, all the other predators stay still for passive besiegement while the one nearest to the prey (the winner in terms of the shortest distance to the prey) takes an active action to chase. Such a behavior can be deemed as coordination based on competition, where the predator nearest from the prey is the "winner" and wins the opportunity to do the capturing task while the rest ones are the "losers" and keep unmoved to do the vigilance.

Consensus algorithms, as modeling of cooperation, update the state by mitigating differences among agents involved, which endow a group of dynamic agents reach an agreement on certain quantities of interest. They have been widely investigated and employed in many distributed problems (Cheng, Hou, & Tan, 2014; Li, Du, & Lin, 2011; Li, Ren, Liu, & Xie, 2013; Li & Zhang, 2010; Seyboth, Dimarogonas, & Johansson, 2013; Wang, Cheng, Ren, Hou, & Tan, 2014). For example, the exact dynamics of agents are generally difficult to obtain, which leads to investigations on the consensus of nonlinear multi-agent systems with unknown dynamics (Chen, Wen, Liu, & Liu, 2016; Liu, Gao, Tong, & Chen, 2016; Liu, Gao, Tong, & Li, 2016; Liu & Tong, 2016). As observed in many fields, competition is of the same importance as cooperation in the emergence of complex behaviors (Li, Zhou, Luo, & You, 2017). However, consensus essentially lacks a mechanism to model competition behaviors, which desires the increase of peer differences and the enhancement of contrasts (Li, Zhou et al., 2017). Therefore, the k-winners-take-all (k-WTA) strategy, which performs the selection of the k competitors whose inputs are larger than the rest ones, has been presented and investigated to describe and capture this competitive nature (Hu & Wang, 2008; Liu & Wang, 2006; Maass, 2000). Author in Maass (2000) prove that a two-layered network composed of weighted averaging in the first layer and WTA in the second layer is able to approximate any nonlinear mapping in any desired accuracy. In addition, it is presented in Li, Zhou et al. (2017) that a k-WTA problem can be equivalently converted to a constrained convex quadratic programming (QP) optimization formulation, which significantly enriches techniques for solving k-WTA problems (Ishizaki et al., 2016; Zhang, Li, Zhang, Luo, & Li, 2015).

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Wei Ren under the direction of Editor Christos G. Cassandras.

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https://doi.org/10.1016/j.automatica.2018.11.001
0005-1098/

Table 1
Comparisons among different control laws for robot control.

	Competitive vs. cooperative	Robot numbers	Distributed vs. centralized	Topology	All connected to command center	Single vs. double integrator model	Neural network involved
This paper	Competitive	Multiple	Distributed	N2N[c]	No	Single[b]	Yes
Paper (Xiao & Zhang, 2014)	NA[a]	Single	NA[a]	NA[a]	NA[a]	Single	No
Paper (Zhang et al., 2015)	Cooperative	Two	Centralized	NA[a]	Yes	Single	Yes
Papers (La, Lim, & Sheng, 2015; Li, Kong, & Guo, 2014; Yoo & Kim, 2015)	Cooperative	Multiple	Distributed	N2N[c]	No	Single	No
Paper (Li, Chen, Liu, Li, & Liang, 2012)	Cooperative	Multiple	Distributed	Star	Yes	Single	Yes
Papers (Jin & Zhang, 2015; Yang, Jiang, Li, & Su, 2017)	Cooperative	Two	Centralized	NA[a]	Yes	Double	Yes

[a] Note that 'NA' means that the item does not apply to the control law presented in the associated papers.

[b] Note that, for the control law proposed in this paper, via some transformation operation, the double-integrator system can be converted into the single-integrator system.

[c] Note that 'N2N' means "Neighbor-to-Neighbor".

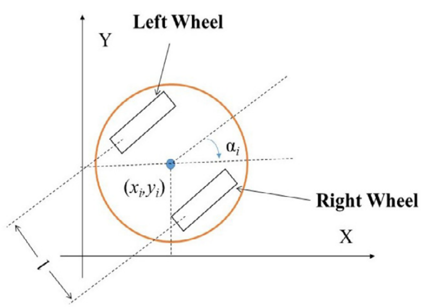

Fig. 1. Differential-driven-wheeled mobile robot model.

Recently, robotics, e.g., multiple mobile robots, have been playing more and more significant roles in scientific researches and engineering applications (La et al., 2015; Li, Chen, Fu, & Sun, 2016; Li, Li, & Kang, 2010; Wang & Gu, 2012; Zhang et al., 2015). In this paper, a new coordination behavior is first defined in a competition manner for tracking a moving target via multiple mobile robots, where only the fittest ones are allocated the task and activated to move towards to the target while the rest ones keep unmoved. Moreover, this is quite different from the existing cooperation control of multiple mobile robots systems, which often requires all mobile robots involved to execute the task together. It is known that, for some situations that the speed of the target is faster than that of trackers, which seems impossible to achieve successful target tracking, a clue from social animals is to besiege the target and capture it by leveraging the bow-on speed. Coinciding with this phenomenon, the proposed distributed coordination also works well when the target has a faster speed. For this situation, the capturing ability of the proposed distributed coordination model with limited communications has also been investigated. It is worth pointing out that, in our previous works (Li, Zhou et al., 2017), we have explored the design of distributed protocols without considering the dynamics of robots, this paper extends them by proposing WTA protocols that can directly lead to proper assignment of tasks among multiple robots for target tracking.

As shown in Table 1, different from the existing control laws for the control of robots, this paper presents a distributed coordination control law based on the competition among multiple mobile robots. Essentially, the existing control laws lack a distributed competitive mechanism to model competition behaviors among the mobile robots involved. As a result, all of the robots in a group aided with these control laws are expected to complete a given task simultaneously without considering the task allocation. It is worth mentioning that, to the best of our knowledge, it is the first time to investigate such a coordination control of mobile robots in a competitive manner. Specifically, the control law proposed in this paper is based on the competitive mechanism rather than the existing cooperation mechanism to achieve a coordination behavior, which is exactly the necessity and meaning of this research. Therefore, the main contributions of this work lie in the proposal of the new competition-based coordinated control behavior and the distributed control law based on competition as well as the corresponding theoretical analyses rather than the traditional cooperation-based coordinated control with the single integrator model.

2. Preliminary and problem formulation

This section presents the preliminary and the problem formulation.

2.1. Differential-driven robot

The differential-driven-wheeled mobile robot shown in Fig. 1 is used to serve as the robot platform in this paper. The kinematic model of the ith differential-driven-wheeled mobile robot is written as

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\alpha}_i \end{bmatrix} = \begin{bmatrix} \dfrac{\cos\alpha_i}{2} & \dfrac{\cos\alpha_i}{2} \\ \dfrac{\sin\alpha_i}{2} & \dfrac{\sin\alpha_i}{2} \\ \dfrac{-1}{l} & \dfrac{1}{l} \end{bmatrix} \begin{bmatrix} \xi_{i1} \\ \xi_{i2} \end{bmatrix}, \tag{1}$$

where (x_i, y_i) denotes the Cartesian coordinates of the middle point of the driving wheel axle; α_i is the bearing of the robot platform with respect to the x-axis, l is the length between the two driving wheels; ξ_{i1} and ξ_{i2} are the speeds of the left and the right wheel, respectively.

Reference to the feedback linearization technique presented in Li et al. (2014), the relationship between the wheel input ξ_i and a transformed input u_i is expressed as

$$\begin{bmatrix} \xi_{i1} \\ \xi_{i2} \end{bmatrix} = \begin{bmatrix} \dfrac{l\sin\alpha_i}{2c} + \cos\alpha_i & \dfrac{-l\cos\alpha_i}{2c} + \sin\alpha_i \\ \dfrac{-l\sin\alpha_i}{2c} + \cos\alpha_i & \dfrac{l\cos\alpha_i}{2c} + \sin\alpha_i \end{bmatrix} \begin{bmatrix} u_{i1} \\ u_{i2} \end{bmatrix},$$

where u_{i1} and u_{i2}, which constitute the vector u_i, denote the control input (the velocity) of the ith robot along X-axis and Y-axis in the new coordinates, respectively; parameters c and l are positive constants. Then, for simplicity as well as for illustration, the motion of each robot could be described by a single integrator:

$$\dot{p}_i = u_i, \tag{2}$$

where $p_i = [x_i + c * \cos(\alpha_i), y_i + c * \sin(\alpha_i)] \in \mathbb{R}^2$ is the position of the new reference position, and locates along the central line of the robot with an offset of c from the wheel center. Please refer to Equation (17) in Li et al. (2014) for detailed derivations of (2).

Remark 1. Via transformation operations, the double-integrator system in the form of $\ddot{x} = u$ can be converted into the single-integrator system in the form of $\dot{y} = v$ described in (2). To see this, consider the following double-integrator system:

$$\ddot{x} = u, \tag{3}$$

and construct the following auxiliary system:

$$\begin{cases} u = -\dot{x} + v, \\ y = \dot{x} + x. \end{cases} \tag{4}$$

Substituting the definition of u into (3) leads to $\ddot{x} = -\dot{x} + v$, which can be further written as $\frac{\mathrm{d}(\dot{x}+x)}{\mathrm{d}t} = v$. Taking into account the definition of y, the above equation can be further converted into a single-integrator system in the form of

$$\dot{y} = v. \tag{5}$$

Therefore, the coordination model based on (2) proposed in this paper can be employed for the coordination control of double-integrator system via the above steps.

2.2. Problem definitions and assumptions

Since the definitions on the communication graph and different communication topologies have been presented in Li et al. (2014), they are not repeated here. In addition, the problem investigated in this paper is defined as follows.

Problem. Under the condition of limited communications, design a coordination model based on competition for n mobile robots described by (1), such that the threshold value w_i of the k fittest mobile robots is 1, thereby enabling these k fittest ones to track the moving target.

2.3. Mathematical symbols and meanings

To lay a basis for further investigation, the mathematical symbols and their corresponding meanings utilized in this paper are listed as follows.

z	Auxiliary variable and can be initialized randomly
λ	$\lambda > 0$
a	Constant being enough small
v	Input to the k-WTA neural network
v_i	The ith element of v
$\bar{v}_k$	The kth largest element in v
$\otimes$	The Kronecker product
I_a	Vector composed of a elements with each one being 1
ρ_i	Estimate of $\frac{1}{n}\sum_{i=1}^{n}\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})$
$\mathbb{N}(i)$	Neighbor set of the ith robot on the communication graph
ϱ_i	Scalar state maintained by the ith mobile robot
A_{ij}	A positive constant for $j \in \mathbb{N}(i)$ with $A_{ij} = A_{ji}$ and for $j \notin \mathbb{N}(i)$, $A_{ij} = A_{ji} = 0$
γ	A positive constant

3. Dynamic task allocation with limited communications

In this section, a distributed competition control law for dynamic task allocation in multi-robot coordination for target tracking with limited communications is presented.

3.1. Model design

To construct the control law, a centralized k-WTA neural network model presented in Hu and Wang (2008) is formulated as follows, which should be modified in the ensuing part to achieve the requirements of distributed control:

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -\lambda\Big(\sum_{i=1}^{n} w_i - k\Big); \tag{6}$$

$$w_i = \mathrm{P}_{\Omega i}\Big(z + \frac{v_i}{2a}\Big), \tag{7}$$

where w_i denotes the ith element of $w \in \{0,1\}^n$, is the threshold value to drive the ith mobile robot; $\mathrm{P}_{\Omega i}(\cdot)$, as the ith element of $\mathrm{P}_{\Omega}(\cdot)$, is defined as $\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a}) = 1$, for $z+\frac{v_i}{2a} > 1$, $\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a}) = z+\frac{v_i}{2a}$, for $0 \le z+\frac{v_i}{2a} \le 1$, and $\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a}) = 0$ for $z+\frac{v_i}{2a} < 0$. Given that $\bar{v}_k$ is strictly larger than $\bar{v}_{k+1}$, and that $a \le 0.5(\bar{v}_k - \bar{v}_{k+1})$, according to Hu and Wang (2008), the above model can solve the following k-WTA problem:

$$w_i = f(v_i) = \begin{cases} 1, & \text{if } v_i \in \{k \text{ largest elements of } v\} \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

In addition, it is defined that $v_i = f_i(z_i) = -\|p_i - p_\mathrm{t}\|_2^2/2$, where p_t denotes the position of the moving target. The movement control for the ith mobile robot is described as

$$\dot{p}_i = w_i c_0 \tau_i, \tag{9}$$

where $c_0 > 0$ and the control law $\tau_i = \partial v_i/\partial p_i$. According to (9), if $w_i = 0$, the ith mobile robot is unmoved, and if $w_i = 1$, the ith mobile robot approaches the moving target.

Remark 2. For constructing an exponentially stable movement control law for the ith mobile robot, Eq. (9) is modified by adding the velocity compensation term:

$$\dot{p}_i = w_i(c_0\tau_i - \dot{p}_\mathrm{t}), \tag{10}$$

where $\dot{p}_\mathrm{t}$ denotes the velocity of the moving target.

Then, Eq. (10) can be formulated as $\dot{p}_i = -w_i(c_0(p_i - p_\mathrm{t}) + \dot{p}_\mathrm{t})$. Letting $e_i = p_i - p_\mathrm{t}$, it can be further derived that $e_i(t) = \exp(-w_i c_0 t)e_i(0)$, with $e_i(0)$ denoting the initial position distance between p_i and p_t, which implies the exponential stability of (10). As long as the value of $\dot{p}_\mathrm{t}$ is bounded, Eq. (9) can be cast into a bounded-input bounded-output (BIBO) system. Given the fact that $\dot{p}_\mathrm{t}$ is bounded, the tracking error synthesized by Eq. (9) is bounded. In addition, the corresponding steady-state tracking error decreases towards zero with the increase of c_0.

Substituting (7) into (6) and (9), it is obtained that

$$\begin{cases} \dot{p}_i = \mathrm{P}_{\Omega i}\Big(z + \dfrac{v_i}{2a}\Big)c_0\dfrac{\partial v_i}{\partial p_i}, \\ \dot{z} = -\lambda\Big\{\displaystyle\sum_{i=1}^{n} \mathrm{P}_{\Omega i}\Big(z + \frac{v_i}{2a}\Big) - k\Big\}. \end{cases} \tag{11}$$

The coordination model can be written in a compact form:

$$\begin{cases} \dot{p} = \mathrm{P}_{\Omega}\Big(zI_{2n} + \dfrac{v}{2a} \otimes I_2\Big)c_0\Phi, \\ \dot{z} = -\lambda\Big(I_n^\mathrm{T}\mathrm{P}_{\Omega}\Big(zI_n + \dfrac{v}{2a}\Big) - k\Big), \end{cases} \tag{12}$$

where $\Phi = [\tau_1, \ldots, \tau_n]^\mathrm{T}$.

In addition, the following theorem can be provided.

Theorem 1. *For a group of n differential-driven robots described by (1) with the coordination control law (12), k robots with the minimum distance move towards the target with time.*

Proof. Define $V_0 = \lambda[\sum_{i=1}^{n} h(z+\frac{v_i}{2a}) - kz]$, where $h(x) = 0$ for $x < 0$; $h(x) = x^2/2$ for $0 \le x \le 1$; and $h(x) = x - 0.5$ for $x > 1$. For the properties of $h(x)$, the following results can be derived.

(1) $\sum_{i=1}^{n}[h(z+\frac{v_i}{2a})-\frac{k(z+v_i/2a)}{n}]$ is lower bounded. It can be concluded that

$$h(x)-\frac{kx}{n}=\begin{cases}-\frac{kx}{n}\geq 0, & \text{if } x<0\\ \frac{x^2}{2}-\frac{kx}{n}\geq -\frac{k^2}{2n^2}, & \text{if } 0\leq x\leq 1\\ -\frac{1}{2}+\frac{n-k}{n}x\geq \frac{n-k}{n}-\frac{1}{2}, & \text{if } x>1.\end{cases}$$

Therefore, $\sum_{i=1}^{n}[h(z+\frac{v_i}{2a})-\frac{k(z+v_i/2a)}{n}]$ is lower bounded.

(2) $\partial h(x)/\partial x = \mathrm{P}_{\Omega}(x)$.

In addition, it is defined that $L_i=-v_i=-f_i(z_i)=\|p_i-p_{\mathrm{t}}\|_2^2/2$. Let $\Upsilon=\frac{2a}{c_0\lambda}V_0+\frac{1}{c_0}\sum_{i=1}^{n}L_i$. Therefore, the properties of Υ and $\dot{\Upsilon}$ also can be derived as follows.

(1) The properties of Υ. Its expression can be given as

$$\Upsilon=\frac{2a}{c_0}\sum_{i=1}^{n}[h(z+\frac{v_i}{2a})-\frac{k}{n}(z+\frac{v_i}{2a})]+\frac{n-k}{c_0 n}\sum_{i=1}^{n}(-v_i).$$

Note that, as proven above, $\sum_{i=1}^{n}[h(z+\frac{v_i}{2a})-\frac{k(z+v_i/2a)}{n}]$ is lower bounded. Besides, $-v_i$ is also lower bounded. Therefore, Υ is lower bounded.

(2) The properties of $\dot{\Upsilon}$. It is derived that $\dot{\Upsilon}=\left(\frac{\partial\Upsilon}{\partial z}\right)^{\mathrm{T}}\dot{z}+\sum_{i=1}^{n}\left(\frac{\partial\Upsilon}{\partial v_i}\right)^{\mathrm{T}}\dot{v}_i$, in which, $\frac{\partial\Upsilon}{\partial z}=\frac{2a}{c_0}\sum_{i=1}^{n}(\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})-\frac{k}{n})=\frac{2a}{c_0}\sum_{i=1}^{n}(w_i-\frac{k}{n})=\frac{2a}{c_0}\sum_{i=1}^{n}w_i-k$, and $\frac{\partial\Upsilon}{\partial v_i}=\frac{2a}{c_0}(\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})\frac{1}{2a}-\frac{k}{n}\frac{1}{2a})+\frac{k}{c_0 n}-\frac{1}{c_0}=\frac{1}{c_0}\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})-\frac{1}{c_0}$; $\dot{v}_i=\frac{\partial f_i(p_i)}{\partial p_i}^{\mathrm{T}}\dot{p}_i=c_0\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})\|\frac{\partial f_i(p_i)}{\partial p_i}\|_2^2$.

It can be further derived that $\left(\frac{\partial\Upsilon}{\partial z}\right)^{\mathrm{T}}\dot{z}=\frac{2a}{c_0}\left(\sum_{i=1}^{n}w_i-k\right)^{\mathrm{T}}\dot{z}=-\lambda\frac{2a}{c_0}\left(\sum_{i=1}^{n}w_i-k\right)^{\mathrm{T}}\left(\sum_{i=1}^{n}w_i-k\right)=-\lambda\frac{2a}{c_0}\left(\sum_{i=1}^{n}w_i-k\right)^2\leq 0$, and $\left(\frac{\partial\Upsilon}{\partial v_i}\right)^{\mathrm{T}}\dot{v}_i=(\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})-1)\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})\|\frac{\partial f_i(p_i)}{\partial p_i}\|_2^2$. It can be concluded from the definition of $\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})$ that $(\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})-1)\leq 0$ and that $\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})\geq 0$. Then, we have $\left(\frac{\partial\Upsilon}{\partial v_i}\right)^{\mathrm{T}}\dot{v}_i\leq 0$, with = holding for $\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})=1$ or $\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})=0$. In addition, it can be obtained readily that $\dot{\Upsilon}\leq 0$.

Using LaSalle's principle and letting $\dot{\Upsilon}=0$, we have, $\forall i$,

$$(\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})-1)\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})\|\frac{\partial f_i(p_i)}{\partial p_i}\|_2^2=0, \tag{13}$$

and

$$\sum_{i=1}^{n}w_i=k. \tag{14}$$

The following results are generalized from (13) and (14).

- As to (13), we have
  - Subcase 1. $\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})=1\Rightarrow w_i=1\Rightarrow \dot{p}_i=c_0\tau_i$, and we have $p_i\to p_{\mathrm{t}}$ as $t\to\infty$.
  - Subcase 2. $\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})=0\Rightarrow w_i=0\Rightarrow \dot{p}_i=0$, and we have that p_i is unmoved.
  - Subcase 3. $\frac{\partial f_i(p_i)}{\partial p_i}=0\Rightarrow p_i=p_{\mathrm{t}}$
- As to (14), it can be obtained that $\sum_{i=1}^{n}w_i=k=\sum_{i=1}^{n}\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})$. Reorder v_i for $i=1,\ldots,n$ as $v_1^*\geq\cdots v_n^*$. Then, we have $\mathrm{P}_{\Omega i}(z+\frac{v_1^*}{2a})\geq\cdots\geq\mathrm{P}_{\Omega i}(z+\frac{v_n^*}{2a})$.
  For $n1+n2+n3=n$, three preconditions are assumed: (a) $\mathrm{P}_{\Omega i}(z+\frac{v_1^*}{2a})=\cdots=\mathrm{P}_{\Omega i}(z+\frac{v_{n1}^*}{2a})=1$; (b) the values of $\mathrm{P}_{\Omega i}(z+\frac{v_{n1+1}^*}{2a}),\ldots,\mathrm{P}_{\Omega i}(z+\frac{v_{n1+n2}^*}{2a})\in(0,1)$; (c) $\mathrm{P}_{\Omega i}(z+\frac{v_{n1+n2+1}^*}{2a})=\cdots=\mathrm{P}_{\Omega i}(z+\frac{v_{n1+n2+n3}^*}{2a})=0$. According to the above three assumptions, the following three subcases are obtained.
  - Subcase 1. For v_i^* with $i\in\{1,\ldots,n1\}$, it can be derived that $w_i=1\Rightarrow\dot{p}_i=c_0\tau_i$, and finally, $p_i\to p_{\mathrm{t}}$ as $t\to\infty$.
  - Subcase 2. For v_i^* with $i\in\{n1+1,\ldots,n1+n2\}$, it can be derived that $w_i>0$, and finally, $p_i\to p_{\mathrm{t}}$ as $t\to\infty$.
  - Subcase 3. For v_i^* with $i\in\{n1+n2+1,\ldots,n1+n2+n3\}$, it can be derived that $w_i=0\Rightarrow\dot{p}_i=0$, and finally, p_i can be unmoved.

  For the first two subcases, v_i goes to the maximal value, and thus $\mathrm{P}_{\Omega i}(z+\frac{v_{n1+1}^*}{2a})$ reaches the same value. Moreover, it can be generalized that $k=\sum_{i=1}^{n}\mathrm{P}_{\Omega i}(z+\frac{v_i^*}{2a})=n1+\sum_{i=n1+1}^{n1+n2}\mathrm{P}_{\Omega i}(z+\frac{v_i^*}{2a})$. For $i\in\{n1+1,\ldots,n1+n2\}$, $\mathrm{P}_{\Omega i}(z+\frac{v_i^*}{2a})=1$. Thus, $k=n1+n2$ and all of them are winners.

Up to this moment, the proof is thus complete. □

Based on the average consensus estimator presented in Freeman, Yang, and Lynch (2006), a mobile robot is able to estimate the average of filter inputs by running the following protocol:

$$\begin{cases}\dot{\rho}_i= & -\gamma\sum_{j\in\mathbb{N}(i)}A_{ij}(\rho_i-\rho_j)-\gamma(\rho_i-\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a}))\\ & -\gamma\sum_{j\in\mathbb{N}(i)}A_{ij}(\varrho_i-\varrho_j),\\ \dot{\varrho}_i= & \sum_{j\in\mathbb{N}(i)}A_{ij}(\rho_i-\rho_j).\end{cases} \tag{15}$$

By running (15) on every mobile robot, ρ_i is able to track $\sum_{i=1}^{n}w_i/n$ or $\sum_{i=1}^{n}\mathrm{P}_{\Omega i}(z+v_i/(2a))/n$. Replacing the term $\sum_{i=1}^{n}\mathrm{P}_{\Omega i}(z+v_i/(2a))$ in (11) with the distributed filter (15) leads to

$$\begin{cases}\dot{\rho}_i= & -\gamma\sum_{j\in\mathbb{N}(i)}A_{ij}(\rho_i-\rho_j)-\gamma(\rho_i-\mathrm{P}_{\Omega i}(z+\frac{v_i}{2a}))\\ & -\gamma\sum_{j\in\mathbb{N}(i)}A_{ij}(\varrho_i-\varrho_j),\\ \dot{\varrho}_i= & \sum_{j\in\mathbb{N}(i)}A_{ij}(\rho_i-\rho_j)\\ \dot{p}_i= & \mathrm{P}_{\Omega i}(z+\frac{v_i}{2a})c_0\frac{\partial v_i}{\partial p_i},\\ \dot{z}= & -\lambda(n\rho_i-k),\end{cases} \tag{16}$$

which can be written in a compact form:

$$\begin{cases}\dot{\rho}= & -\gamma L\rho-\gamma(\rho-w)-\gamma L\int_{t_0}^{t}L\rho\,\mathrm{d}t,\\ \dot{p}= & \mathrm{P}_{\Omega}(zI_{2n}+\frac{v}{2a}\otimes I_2)c_0\Phi,\\ \dot{z}= & -\lambda(I_n^{\mathrm{T}}\rho-k),\end{cases} \tag{17}$$

where t_0 stands for the initial time instant; Laplacian matrix $L=\mathrm{diag}(AI_n)-A$.

As stated in Yang et al. (2010), the distributed consensus filter (15) has several advantages compared with other existing methods. For example, given that the network is connected, the estimator error converges to a ball around zero with the radius related to the rate of change of the input. In the situation of constant input, the corresponding estimator error converges exponentially to zero. Therefore, the condition for the communication topology to solve the distributed k-WTA problem aided with mobile robots is that the communication network formed by mobile robots is connected.

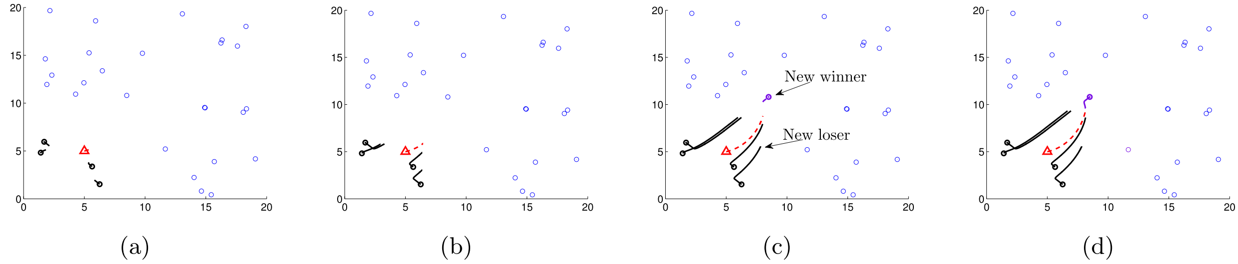

Fig. 2. Snapshots for moving target tracking and the corresponding tracking trajectories, where initial locations of mobile robots and the moving target are randomly generated. (a) Snapshots at $t = 0.3$ s. (b) Snapshots at $t = 1.7$ s. (c) Snapshots at $t = 2.5$ s. (d) Actual path of the moving target and tracking trajectories of different mobile robots.

Remark 3. If the parameter γ is large enough relative to λ, then it can be expected that the resulting dynamics converges semiglobally (Yang et al., 2010).

Remark 4. A potential and fundamental assumption for target tracking via mobile robots is that the speed of the target should be slower than that of the mobile robots. However, in real life, the situation that the speed of the target is faster than that of trackers may exist, and an effective way to fix such a knotty problem is to besiege the target and try to capture the target by leveraging the bow-on speed. Inspired by such a behavior existing widely, the capturing ability of the distributed coordination model (17) with limited communications in this situation is investigated. A method for simulating such a behavior is to employ the following saturation function with $\dot{p}_i^+$ and $\dot{p}_i^-$ denoting the upper and lower limits of the ith mobile robot, respectively:

$$\mathbb{S}(\dot{p}_i) = \begin{cases} \dot{p}_i^+, & \text{if } \dot{p}_i > \dot{p}_i^+ \\ \dot{p}_i, & \text{if } \dot{p}_i^- \leq \dot{p}_i \leq \dot{p}_i^+ \\ \dot{p}_i^-, & \text{if } \dot{p}_i < \dot{p}_i^-. \end{cases}$$

4. Illustrative examples

In this section, the parameters are set as follows: $n = 30$, $k = 4$, $a = 0.1$, $c_0 = 10$, $\lambda = 10$, $\gamma = 10^5$, the distance tolerant $\varepsilon = 0.01$ m, $\dot{p}_i^+ = -\dot{p}_i^- = 1.8$ m/s. In addition, $A_{ij} = 1$ for $|i - j| \leq 1$, otherwise, $A_{ij} = 0$.

In this example, as the pursuer, each mobile robot is not able to go after the target and to capture it directly because the speed of the moving target can be faster than that of the mobile robots. The corresponding simulation results, are illustrated in Figs. 2 and 3.

As shown in Fig. 2(a), at $t = 0.3$ s, the position of the moving target is around (5, 5), from which the nearest mobile robots marked in black lines win the competition and can be deemed as being allocated the tracking task. In comparison, the rest ones, as the losers of the competition, are deactivated and unmoved. As a result, the winner begins to track the moving target. However, according to Fig. 2(b), it can be seen that the speed of the moving target can be faster than that of the mobile robots, these winner mobile robots cannot capture it. In addition, as the moving target approaches one of the losers, one of the winners at the initial time fails in the competition and becomes a loser afterward (see Fig. 2(c)). As a continuator, the new winner marked in purple line begins to track the moving target head-on. Moreover, Fig. 2(d) illustrates the corresponding actual path of the moving target as well as the tracking trajectories, which shows that the trajectories of the mobile robots are smooth. In addition, as the losers, the rest mobile robots keep unmoved. These simulation results verify preliminarily the effectiveness of the proposed distributed coordination model (17) with limitation on the speed of each mobile robot and limited communications.

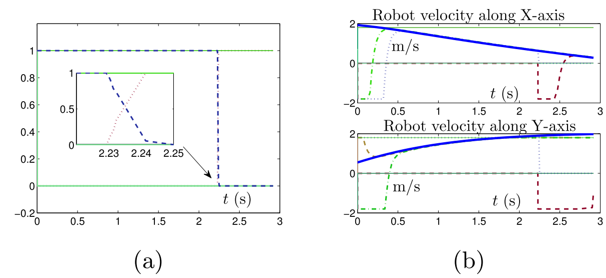

Fig. 3. Outputs of k-WTA network and the corresponding velocities of mobile robots at different phases. (a) Outputs of k-WTA network. (b) Velocity profiles of mobile robots with blue lines denoting the velocity profiles of the moving target. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

It can be found in Fig. 3(a) that the outputs of the k-WTA network rapidly converge to the correct results. The velocities of the mobile robots are shown in Fig. 3(b), from which, it can be observed that the velocities of winner mobile robots are bounded by the given limits. The value of the velocity of the moving target can be 2 m/s, which is evidently faster than that of the mobile robots. However, similar to the situations existing in real world, such a knotty problem can be handled by besieging the target and then capturing it. These results further verify the effectiveness of the proposed distributed coordination model (17) with limitation on the speed of each mobile robot and limited communications.

5. Conclusion and future research directions

In this paper, a new coordination behavior has been defined in a competitive manner for task allocation in tracking a moving target via multiple mobile robots, in which only the winners of the competition are activated to move towards the target while the rest ones keep unmoved. A distributed coordination model with limited communications and with the aid of a distributed consensus filter has been proposed as well. The stability of the distributed control has been proved in theory. In addition, since the speed of the target may be faster than that of mobile robots, the target tracking task with speed limitations on mobile robots has been investigated via the proposed model.

It is worth mentioning that the WTA index υ in (17) can be chosen as other forms of measurements, e.g., the combination forms of relative position and relative velocity, which would be future research directions. In addition, another one of the valuable future research directions would be the competitive control among robots at the robot dynamic level with force as the control input.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (with No. 61703189), by the Opening Foundation of Key Laboratory of Opto-technology and Intelligent Control (Lanzhou Jiaotong University), Ministry of Education, China (No. KFKT2018-1), by the Natural Science Foundation of Gansu Province, China (No. 18JR3RA264), by the National Natural Science Foundation of China (No. 11561029), by the National Basic Research Program of China (973 Program) (No. 2014CB744600), by the Belt and Road Special Project of Lanzhou University, China (No. 2018ldbryb020), by the Fundamental Research Funds for the Central Universities, China (No. lzujbky-2017-37), by the Natural Science Foundation of Hunan Province, China, (No. 2017JJ3257), by the Research Foundation of Education Bureau of Hunan Province, China (No. 17C1299), by Hong Kong Research Grants Council Early Career Scheme (with No. 25214015), and also by Hong Kong Polytechnic University (with numbers G-YBMU, G-UA7L, 4-ZZHD, F-PP2C, 4-BCCS).

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