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Flocking of Mobile Robots by Bounded Feedback

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***Abstract*—This paper addresses the problem of flocking of mobile robots by bounded feedback. A control problem is formulated with the multiple goals: velocity consensus, collision avoidance, and cohesion maintenance. The flocking protocol is then obtained by means of control design. Particularly, a Lyapunov-like function is presented, the velocity control is designed to achieve coordination and linear speed consensus, and the steering control is designed for consensus on orientation. The convergence is verified via Barbalat's lemma. Simulation results with a group of mobile robots are presented to show the effectiveness of the proposed flocking control.**

I. INTRODUCTION

Flocking is the collective coherent motion of self-propelled organisms [1]. For its wide range of engineering applications, flocking of mobile agents has been an active research area [2]–[5]. A fundamental issue is to obtain the flocking protocol under which a desired collective motion emerges. The research is transforming from empirical protocols for agents of point-mass models to systematically designed protocols for agents of actual physical models, or for agents with rectangular shapes [6]. Since [7]–[12], significant effort has been made for flocking of mobile robots [13], [14]. Recently, a measure-theoretic approach for systematic design obtaining flocking protocol for mobile robots has been presented [15]. In this paper, we address the further problem of flocking of mobile robots by bounded feedback, which is motivated by the implementation issue that the motor speed cannot be as large as desired for the limited electric current.

We are interested in the full dynamic model of mobile robot derived in [16]. Due to nonholonomic nature, we shall achieve velocity consensus through a modular design framework, which separately achieves consensus on the linear speed v_i and consensus on the orientation angles θ_i. Since the orientation angle is a kinematic variable whose rate of change is not matched to the input controlling the dynamic variable, consensus on orientation angle by bounded feedback is a challenging problem, which calls for a dedicated design approach. Addressing this issue, the current paper contributes a method for orientation consensus of mobile robots.

On the other hand, for engineering applications, cohesion maintenance and collision avoidance (CMCA) are necessary. The works [13], [17] indicate that the attractive and repulsive forces cannot be included in the control for CMCA of mobile robots, as it is possible for point-mass agents [10]. In [15], a rearrangement strategy is presented for inducing desired attractive and repulsive forces for CMCA of mobile robots. In this paper, due to the boundedness constraint, we redesign the coordination function such that the induced attractive and repulsive forces are bounded, and hence can be included in the bounded velocity control. It turns out that the maximal value of the coordination function determines the basin of attraction for the flocking convergence.

In the context of the current paper, we assume all-to-all communication among agents to present our design procedure without invoking graph theory as in the case of nearest neighbor communication [9], [10]. Also, we are interested in the plain velocity consensus without specifying the desired heading direction. These simplifying conditions allow us to focus on the introduction of our bounded control design.

The rest of the paper is organized as follows. In Section II, we formulate the multiple-goal control problem for flocking of mobile robots. In Section III, a modular design framework is presented for bounded velocity control and bounded orientation control. The theoretical results are also introduced in Section III. The simulation results are presented in Section IV. The paper is concluded with Section V.

Notations: $\mathbb{R}$ and $\mathbb{R}^+$ are the sets of real numbers and non-negative real numbers, respectively; for $q = [q_1, \ldots, q_n]^T$, $\nabla_q = [\partial/\partial q_1, \ldots, \partial/\partial q_n]^T$ is the del operator [18]; for two vectors a and b, $a \cdot b$ is their scalar product; $(a_1, \ldots, a_n)$ is $[a_1^T, \ldots, a_n^T]^T$; $|\cdot|$ is the absolute value of scalars; and $\|\cdot\|$ is the Euclidean norm of vectors.

II. PROBLEM FORMULATION

Given a collective system of N identical autonomous mobile robots labeled by the numbers $1, ..., N$ whose respective equations of motion are [16]

$$\begin{aligned}\dot{q}_i &= v_i e(\theta_i)\\ \dot{\theta}_i &= w_i\\ \dot{v}_i &= u_i\\ \dot{w}_i &= \tau_i\end{aligned} \tag{1}$$

where, as shown in Fig. 1, $q_i = [x_i, y_i]^T \in \mathbb{R}^2$, and $\theta_i \in \mathbb{R}$ are respectively the position and the heading angle of the i-th robot in the inertial frame Oxy; $v_i \in \mathbb{R}$ is the linear speed, and $e(\theta_i)$ the unit vector $[\cos(\theta_i, \sin\theta_i]^T$; $w_i \in \mathbb{R}$ is the angular speed, and $u_i, \tau_i \in \mathbb{R}$ are control inputs mentioned as speed control and steering control, respectively.

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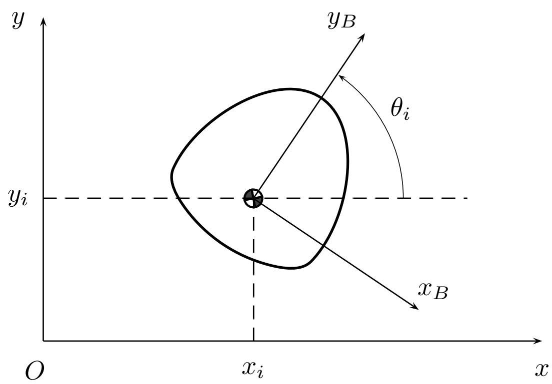

Fig. 1. Configuration of the i-th mobile robot.

Let r_0, R_0 be positive constants, $r_0 < R_0$. Our flocking control problem for (1) is to obtain the controls u_i, τ_i as bounded functions of the collective state $(q_1, \ldots, q_N, \theta_1, \ldots, \theta_N, v_1, \ldots, v_N, w_1, \ldots, w_N)$ such that the following multiple goals are achieved

G1) *Velocity consensus:*

$$\lim_{t\to\infty} (\dot{q}_i(t) - \dot{q}_j(t)) = 0, \forall i, j = 1, \ldots, N \qquad (2)$$

G2) *Collision avoidance:* $r_{ij}(t) = \|q_i(t) - q_j(t)\| \geq r_0, \forall t \geq 0, \forall i \neq j$

G3) *Cohesion maintenance:* $r_{ij}(t) \leq R_0, \forall t \geq 0, \forall i \neq j$.

For disambiguation, we have the following definition.

Definition 2.1: A control $\dot{\zeta} = g(\zeta, y), u = c(\zeta, y), (\zeta, y) \in \mathbb{R}^d \times \mathbb{R}^m$ of a system $\dot{x} = f(x, u), y = h(x, u)$ is said to be bounded if there is a finite constant $M > 0$ such that $\|c(\zeta, y)\| \leq M, \forall (\zeta, y) \in \mathbb{R}^d \times \mathbb{R}^m$.

To achieve the goals G2) and G3), we consider the coordination function

$$U_1(t) = \frac{1}{2N} \sum_{i=1}^{N} \sum_{j=1}^{N} U(r_{ij}(t)) \qquad (3)$$

where $U : \mathbb{R}^+ \to \mathbb{R}^+$ is a function satisfying

P1) there are positive constants U_L and U_R and an $r \in [r_0, R_0]$ such that

$$\begin{aligned} &0 \leq U(r) \leq U_L, \forall r \leq a; \text{and} \\ &0 \leq U(r) \leq U_R, \forall r \geq a \end{aligned} \qquad (4)$$

P2) $U(r)$ is continuously differentiable on $[r_0, R_0]$;

P3) $\lim_{r\to r_0^+} U(r) = U_L$; and

P4) $\lim_{r\to R_0^-} U(r) = U_R$.

Since we are maintaining $r_{ij}(t) \in [r_0, R_0]$ for $i \neq j$, without loss of generality, we assume that $U(0) = 0$ and hence (3) is well defined for $r_{ii} = 0$.

A function U satisfied the above properties is depicted in Fig. 2. We are interested in the function U with the dead zone $[a, A]$ for free alignment, that mimics the orientation zone in animal flocking [19].

Clearly, by maintaining $2NU_1 < U_L$, we have $U(r_{ij}(t)) < U_L, \forall t$, implying that $r_{ij}(t) \geq r_0, \forall t$. If, in addition, we have $U_1(0) < NU_R$, then $r_{ij}(t) \leq R_0, \forall t$. Accordingly, the goals G2) and G3) are achieved.

For bounded control, we shall use the linear saturation functions σ_1, σ_2 and σ_3, which are continuous and nondecreasing functions and satisfy, for given positive constants $L_i \leq M_i, i = 1, 2, 3$:

i) $\sigma_i(-s) = -\sigma_i(s)$ for all s;
ii) $\sigma_i(s) = s$ for $s \leq L_i$; and
iii) $|\sigma_i(s)| \leq M_i, \forall s \in \mathbb{R}$.

For bounded backstepping, we shall use the scaling function Ω [20], which is a real-valued and continuously differentiable and satisfies, for a positive constant B,

Ω1) $\Omega(s) = s, \forall s \in [-2B, 2B]$; and
Ω2) $\Omega(-s) = -\Omega(s), \Omega'(s) \geq 1, \forall s$.

We end this section with the following lemmas.

Lemma 2.1: Let $\sigma : \mathbb{R} \to \mathbb{R}$ be a function satisfying $\sigma(-s) = -\sigma(s), \forall s \in \mathbb{R}$. Then, for all a_i, b_i, it holds true that

$$\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (a_i - a_j)\sigma(b_i - b_j) = \sum_{i=1}^{N} \sum_{j=1}^{N} a_i \sigma(b_i - b_j). \qquad (5)$$

Proof: As $\sigma(-s) = -\sigma(s)$, we have

$$\begin{aligned} \sum_{i=1}^{N} \sum_{j=1}^{N} a_j \sigma(b_i - b_j) &= -\sum_{i=1}^{N} \sum_{j=1}^{N} a_j \sigma(b_j - b_i) \\ &= -\sum_{i=1}^{N} \sum_{j=1}^{N} a_i \sigma(b_i - b_j). \end{aligned} \qquad (6)$$

Hence,

$$\begin{aligned} &\sum_{i=1}^{N} \sum_{j=1}^{N} (a_i - a_j)\sigma(b_i - b_j) \\ &= \sum_{i=1}^{N} \sum_{j=1}^{N} a_i \sigma(b_i - b_j) - \sum_{i=1}^{N} \sum_{j=1}^{N} a_j \sigma(b_i - b_j) \\ &= 2 \sum_{i=1}^{N} \sum_{j=1}^{N} a_i \sigma(b_i - b_j) \end{aligned} \qquad (7)$$

which implies (5). ■

Lemma 2.2: The linear saturation functions $\sigma_i, i = 1, 2, 3$ satisfy

$$(\sigma_i(\theta_1) - \sigma_i(\theta_2))\sigma_i(\theta_1 - \theta_2) \geq 0, \forall \theta_1, \theta_2. \qquad (8)$$

Proof: Without loss of generality, suppose that $\theta_1 \geq \theta_2$. Since σ_i are nondecreasing functions, this implies that

$$\sigma_i(\theta_1) - \sigma_i(\theta_2) \geq 0. \qquad (9)$$

Furthermore, as $\sigma_i(0) = 0$, $\theta_1 \geq \theta_2$ and the nondecreasing property of σ_i imply that

$$\sigma_i(\theta_1 - \theta_2) \geq 0. \qquad (10)$$

Multiplying (9) and (10) side-by-side, we obtain (8). ■

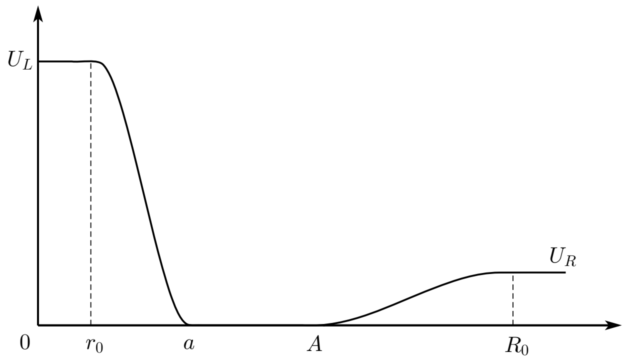

Fig. 2. Coordination function.

III. PROTOCOL DESIGN

Our design strategy is to design u_i to achieve consensus on v_i, and design τ_i to achieve consensus on θ_i. As τ_i is not the direct input of θ_i dynamics, we shall develop a backstepping procedure based on [20].

Since $U(r_{ij}) = U(\|q_i - q_j\|)$, in the following, we shall consider U as the symmetric function of q_i and q_j as well, and we write $U(q_i, q_j)$ with the understanding that $U(q_i, q_j) = U(q_j, q_i)$. For this symmetry, we have

$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\dot{U}(q_i, q_j) = \sum_{i=1}^{N}\sum_{j=1}^{N}\nabla_{q_i}U(q_i, q_j)\cdot q_i. \tag{11}$$

Our design is Lyapunov-based. Particularly, we shall construct a positive definite function V and solve for the protocols u_i and τ_i such that the time derivative of V contains desired dissipation terms. Then, we apply Barbalat's lemma [21] to conclude the desired consensuses.

The construction of such function V is made modular as follows.

A. Speed consensus

Consider the function

$$V_1 = U_1 + \frac{1}{2}\sum_{i=1}^{N} v_i^2. \tag{12}$$

Using (1) and (11), we compute

$$\begin{aligned}\dot{V}_1 &= \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\nabla_{q_i}U(q_i, q_j)\cdot\dot{q}_i + \sum_{i=1}^{N} v_i u_i \\ &= \frac{1}{N}\sum_{i=1}^{N} v_i\left(\sum_{j=1}^{N}\nabla_{q_i}U(q_i, q_j)\cdot e(\theta_i)\right) + \sum_{i=1}^{N} v_i u_i.\end{aligned} \tag{13}$$

From (13), we have the following design for speed consensus protocol

$$u_i = -\frac{1}{N}\sum_{j=1}^{N}\nabla_{q_i}U(q_i, q_j)\cdot e(\theta_i) - \frac{1}{N}\sum_{i=1}^{N}\sigma_1(v_i - v_j) \tag{14}$$

where σ_1 is the linear saturation function introduced in Section II.

Substituting (14) into (13), we obtain

$$\dot{V}_1 = -\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} v_i\sigma_1(v_i - v_j). \tag{15}$$

We have the following speed consensus theorem.

Theorem 3.1: Suppose that the collective system (1) subject to the protocol (14) is initiated such that $2NV_1 < U_M$. Then, the following properties hold true:

i) $U(q_i(t), q_j(t)) \le U_L, \forall t, \forall i \neq j$; and
ii) $\lim_{t\to\infty}(v_i(t) - v_j(t)) = 0$

Proof: As designed, (15) holds true. By Lemma 2.1, (15) yields,

$$\dot{V}_1 = -\frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}(v_i - v_j)\sigma_1(v_i - v_j). \tag{16}$$

Since σ_1 is a linear saturation function, the right-hand-side of (16) is negative definite. Accordingly, $V_1(t)$ is nonincreasing in t, and we have

$$U_L > 2NV_1(t) \ge 2NU_1 \ge U(q_i(t), q_j(t)), \forall t, \forall i, j \tag{17}$$

which verifies the conclusion i).

Furthermore, as $V_1(t)$ is nonincreasing, a standard application of Barbalat's lemma [21] to (16) indicates that the right-hand-side of (16) converges to zero, which verifies the conclusion ii) of the theorem. ■

By Theorem 3.1, the design (14) achieves speed consensus and the goals G2) and G3). We have the following subsection designing steering laws τ_i for orientation consensus completing the goal G1).

B. Orientation Consensus

Since the dynamics of θ_i is a double integrator, we shall develop a bounded backstepping approach which is motivated by the result [20] for single nonlinear systems.

Consider the function

$$V_2 = \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}\int_0^{\theta_i - \theta_j}\sigma_2(s)ds. \tag{18}$$

We compute

$$\begin{aligned}\dot{V}_2 &= \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_2(\theta_i - \theta_j)(w_i - w_j) \\ &= \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_2(\tilde{\theta}_{ij})(w_i^* - w_j^*) \\ &\quad + \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_2(\tilde{\theta}_{ij})(w_i - w_i^* - (w_j - w_j^*))\end{aligned} \tag{19}$$

where $\tilde{\theta}_{ij} = \theta_i - \theta_j$, and w_i^* is the so-called virtual control to be specified.

Consider the virtual protocol

$$w_i^* = -\frac{1}{N}\sum_{j=1}^{N}(\sigma_2(\theta_i) - \sigma_2(\theta_j)) = -\sigma_2(\theta_i) + \bar{\sigma}_2 \quad (20)$$

where

$$\bar{\sigma}_2 = \frac{1}{N}\sum_{j=1}^{N}\sigma_2(\theta_j). \quad (21)$$

Note that

$$w_i^* - w_j^* = -(\sigma_2(\theta_i) - \sigma_2(\theta_j)). \quad (22)$$

Substituting (20) and (22) into (19) and using Lemma 2.1, we obtain

$$\begin{aligned}\dot{V}_2 &= -\frac{1}{2N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_2(\tilde{\theta}_{ij})(\sigma_2(\theta_i) - \sigma_2(\theta_j)) \\ &\quad + \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_2(\tilde{\theta}_{ij})\Big(w_i + \sigma_2(\theta_i) - (w_j + \sigma_2(\theta_j))\Big) \\ &= -\frac{1}{2N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_2(\theta_i - \theta_j)(\sigma_2(\theta_i) - \sigma_2(\theta_j)) \\ &\quad + \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_2(\tilde{\theta}_{ij})\Big(w_i + \sigma_2(\theta_i)\Big). \quad (23)\end{aligned}$$

By Lemma 2.2, the first term in the second equation of (23) is negative definitive, which is desired for the orientation consensus. The second term is to be canceled. To this end, we augment V_2 to obtain the function

$$V_3 = V_2 + \frac{1}{2}\sum_{i=1}^{N}\Big(\Omega(w_i) + \sigma_2(\theta_i)\Big)^2 \quad (24)$$

where Ω is the scaling function Ω introduced in Section II.

To design the control τ_i bounded, let us define the variables

$$\begin{aligned}\xi_i &= w_i + \sigma_2(\theta_i) \\ \Omega_i &= \Omega(w_i) + \sigma_2(\theta_i). \quad (25)\end{aligned}$$

We have

$$\dot{\Omega}_i = \Omega'(w_i)\tau_i + \sigma_2'(\theta_i)w_i \quad (26)$$

For brevity, define

$$\tilde{\sigma}_2(\theta_{ij}) = \sigma_2(\theta_i) - \sigma_2(\theta_j). \quad (27)$$

From (24) and (23), we have

$$\begin{aligned}\dot{V}_3 &= -\frac{1}{2N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_2(\tilde{\theta}_{ij})\tilde{\sigma}_2(\theta_{ij}) + \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_2(\tilde{\theta}_{ij})\xi_i \\ &\quad + \sum_{i=1}^{N}\Omega_i\Big(\Omega'(w_i)\tau_i + \sigma_2'(\theta_i)w_i\Big) \\ &= -\frac{1}{2N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_2(\tilde{\theta}_{ij})\tilde{\sigma}_2(\theta_{ij}) + \sum_{i=1}^{N}\xi_i\Bigg(\frac{\Omega_i}{\xi_i}\Big(\Omega'(w_i)\tau_i \\ &\quad + \sigma_2'(\theta_i)w_i\Big) + \frac{1}{N}\sum_{j=1}^{N}\sigma_2(\tilde{\theta}_{ij})\Bigg). \quad (28)\end{aligned}$$

Using (28), we have the following design for bounded τ_i

$$\begin{aligned}\tau_i &= -\frac{1}{N}\frac{\xi_i}{\Omega_i}\frac{1}{\Omega_i'}\Bigg(\sum_{j=1}^{N}\sigma_3(\xi_i - \xi_j) + \sum_{j=1}^{N}\sigma_2(\tilde{\theta}_{ij})\Bigg) \\ &\quad - \frac{w_i}{\Omega_i'}\sigma_2'(\theta_i). \quad (29)\end{aligned}$$

We now verify that τ_i is well-defined. Indeed, by property Ω1) of the function Ω introduced in Section II, we have

$$\Omega(w_i) = w_i, \text{ for } w_i \in [-2B, 2B]. \quad (30)$$

Accordingly, in view of (25), for $w_i \in [-2B, 2B]$, we have

$$\frac{\xi_i}{\Omega_i} = 1. \quad (31)$$

For $w_i \notin [-2B, 2B]$, we have $|\Omega(w_i)| > 2B$. Accordingly, choosing the saturating value M_2 of the function σ_2 satisfying

$$M_2 < 2B \quad (32)$$

we have

$$\left|\frac{1}{N}\sum_{j=1}^{N}\sigma_2(\theta_i - \theta_j)\right| < 2B \quad (33)$$

which implies that $|\Omega_i| > 0$ for $w_i \notin [-2B, 2B]$, and hence ξ_i/Ω_i is well defined.

Furthermore, as $|\Omega(s)| \geq |s|, \forall s$, the design (33) guarantees that $|\xi_i| \leq |\Omega_i|$ for $w_i \notin [-2B, 2B]$. This and (31) indicate that

$$\left|\frac{\xi_i}{\Omega_i}\right| \leq 1, \forall i. \quad (34)$$

Also, the property Ω2) of the function Ω guarantees that $1/\Omega'(s) \leq 1, \forall s$. This and (34) indicate that the steering law (29) is well-defined.

Substituting (29) into (28) and using (23), we arrive at

$$\begin{aligned}\dot{V}_3 &= -\frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}(\sigma_2(\theta_i) - \sigma_2(\theta_j))\sigma_2(\theta_i - \theta_j) \\ &\quad - \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i\sigma_3(\xi_i - \xi_j). \quad (35)\end{aligned}$$

Applying Lemma 2.1 to the last term of (35), we obtain

$$\begin{aligned}\dot{V}_3 &= -\frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}(\sigma_2(\theta_i) - \sigma_2(\theta_j))\sigma_2(\theta_i - \theta_j) \\ &\quad - \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}(\xi_i - \xi_j)\sigma_3(\xi_i - \xi_j). \quad (36)\end{aligned}$$

We have the following orientation consensus theorem.

Theorem 3.2: Suppose that the collective system (1) is subject to the protocol (29). Then, all the mobile robots eventually reach consensus on the heading angles θ_i, i.e.,

$$\lim_{t\to\infty}(\theta_i(t) - \theta_j(t)) = 0, \forall i, j. \quad (37)$$

Proof: By Lemma 2.2, the right-hand-side of (36) is negative definite. Hence, $V_3(t)$ is nonincreasing, a standard application of Barbalat's lemma [21] to (36) indicates that the right-hand-side of (16) converges to zero, which verifies the conclusion of the theorem. ■

Combining Theorems 3.1 and 3.2, we have the following bounded flocking theorem.

Theorem 3.3: Suppose that the collective system (1) is subject to the bounded protocols (14) and (29). Suppose further that the initial configuration of the collective system (1) is such that $2NV_1(0) < U_L$, $NU_R \le V_1(0)$, and the design parameters satisfy (32). Then, all the multiple flocking goals of velocity consensus, cohesion maintenance, and collision avoidance are achieved.

Proof: Directly combine proofs of Theorems 3.1 and 3.2. ■

IV. SIMULATION

We run simulation for a multi-agent system of 10 mobile robots of the model (1). We used bump function to generate the smooth coordination function U of the shape depicted in Fig. 2. As the control (14) invokes the gradient forces $\nabla_{q_i} U$, we designed the coordination function in the form

$$U(r) = \int_0^r \varphi(s) ds \tag{38}$$

where φ is a compact support function given by

$$\varphi(s) = \begin{cases} p_1 \exp\left(\frac{-(s-s_0)^2}{((a-r_0)/2)^2-(s-s_0)^2}\right) & \text{if } s \in (r_0, a) \\ p_2 \exp\left(\frac{-(s-s_1)^2}{((R_0-A)/2^2-(s-s_1)^2}\right) & \text{if } s \in (A, R_0) \\ 0 & \text{otherwise} \end{cases}$$
$$s_0 = \frac{r_0 + a}{2}$$
$$s_1 = \frac{A + R_0}{2} \tag{39}$$

where a, A, r_0 and R_0 are design parameters, and p_1 and p_2 are computed such that, given U_L, U_R, $U(r_0) = U_L$ and $U(R_0) = U_R$.

We obtained the simulation results shown in Fig. 3–Fig. 7, with $r_0 = 2, a = 3.7, A = 4, R_0 = 8, U_L = 85$, and $U_R = 1$. The arrays $[x_1, \dots, x_{10}]$, $[y_1, \dots, y_{10}]$, and $[\theta_1, \dots, \theta_{10}]$ of the initial positions of the 10 robots are $[-2.2, -10, -3, 1, 7, 1, -8, -4, 3, -7]$, $[-2.2, 6, 1, -4, -1, 2, -3, 5, 8, 2]$, and $[4.7, 1.6, 3.2, 4.4, 5.6, 6, 3.4, 0.8, 0.9, 1.6]$, respectively. All the velocities v_i and angular speeds w_i are initialized at 0.

The flocking behavior is shown in Fig. 3. We observed that no collision occurred. The mobile robots can move and quickly change their headings to flock together. By Fig. 4 and Fig. 5, we conclude that consensuses on orientation and speed of the mobile robots have been obtained. Fig. 6 and Fig. 7 display bounded speed and steering controls.

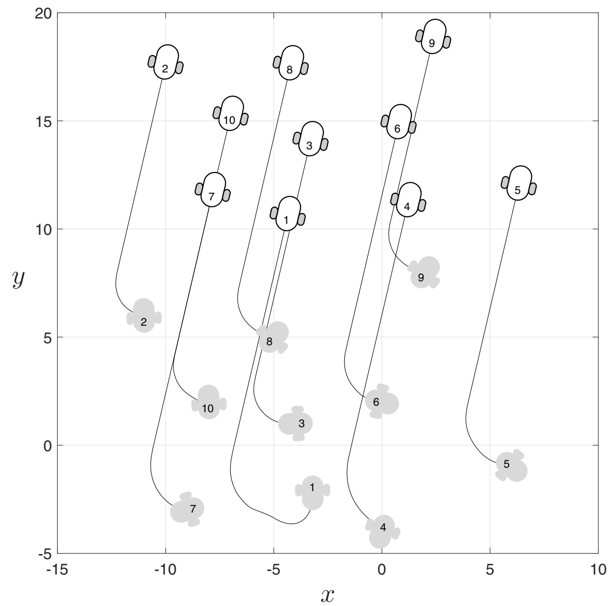

Fig. 3. Flocking of 10 mobile robots.

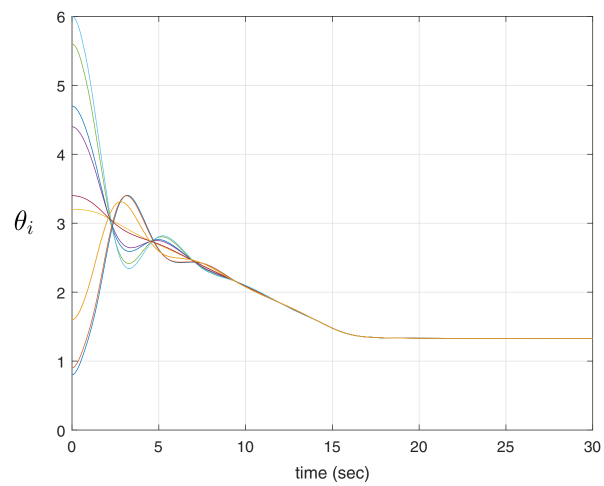

Fig. 4. Orientation consensus.

V. CONCLUSIONS

Bounded protocol for flocking of mobile robots has been obtained by systematic design. Theoretical and simulation results verified that the designed protocol achieves all the multiple goals of the flocking control: velocity consensus, cohesion maintenance, and collision avoidance. Assuming all-to-all communication and without desired orientation, we presented technique for bounded flocking control of mobile robots. The issues of limited communication and forcing the system to follow a virtual leader are open for future research.

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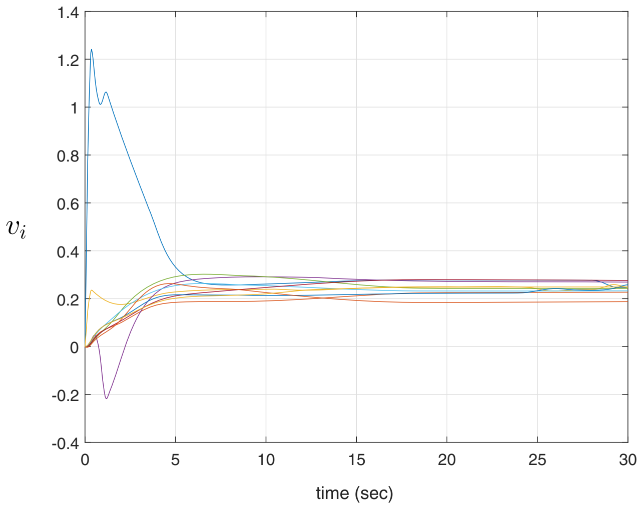

Fig. 5. Speed consensus.

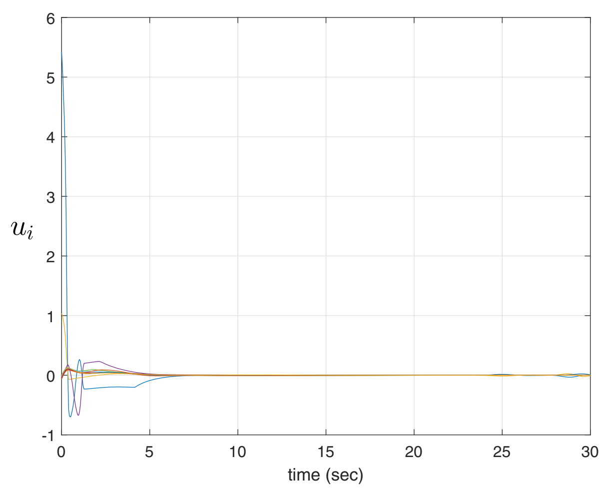

Fig. 6. Speed controls.

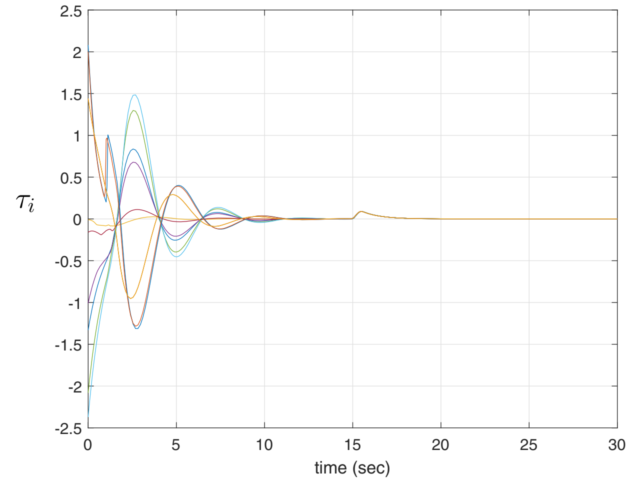

Fig. 7. Steering controls.

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