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Distributed Flocking Control of Mobile Robots by Bounded Feedback

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Abstract— **Flocking control of multiple agents with their point-mass models has been extensively studied. However, flocking control of mobile robots with full dynamic models is challenging research due to nonholonomic nature. This paper presents a novel approach to distributed flocking control of nonholonomic mobile robots by bounded feedback. The flocking control objectives include velocity consensus, collision avoidance, and cohesion maintenance among mobile robots. A flocking protocol which is based on the neighborhood information of mobile robots is constructed by means of control design. A Lyapunov-like function and graph theory are employed for convergence analysis. Simulation results are presented to illustrate the effectiveness of the proposed distributed flocking control scheme.**

Keywords: Flocking control; Multi-agent systems; Multi-robot systems; Cooperative control; Mobile robots; Decentralized control.

I. INTRODUCTION

Flocking control of multiple agents with their point-mass models has been studied for a decade starting from the work of Olfati-Saber [1]. Later, extensive research in flocking control of mobile robots has been carried out for a wide range of engineering applications [2]–[6]. A common objective is to obtain a desired collective motion which can be produced by a constructive flocking protocol. Systematically designed prototols have been proposed for multi-agent systems whose models are described from simplest models such as point-mass models to actual physical models [7]. Significant efforts have been made for studying flocking of mobile robots in [8]–[14] and references therein. Recently, a measure-theoretic approach for systematic design obtaining flocking protocol for mobile robots has been presented [5]. All-to-all communication is assumed in many works, which requires the knowledge of information of all agents [15]. This centralized communication leads to inflexibility and large computation costs for the controller for each agent. Meanwhile, a distributed protocol can offer an ease of implementation, and less burden of computation as it only requires the information of neighbor agents for an element of the system. There have been a range of papers addressing the decentralized control of mobile robots [1], [8], [16], [17]. In this paper, we are interested in addressing the problem of distributed flocking control of mobile robots by bounded feedback, which takes into account nonholonomic nature of mobile robots as well as the implementation issue posed by the physical limit of the motor speed.

In this paper, the full dynamic model of mobile robot derived in [18] is employed for our flocking control problem. Due to nonholonomic nature, a modular design framework is constructed to achieve velocity consensus, in which consensus on the linear speed and consensus on the orientation angles are obtained separately.

It should be noted that cohesion maintenance and collision avoidance (CMCA) are of importance for engineering applications. As pointed out in [12], [19], the attractive and repulsive forces cannot be included in the control for CMCA of mobile robots, as it is possible for point-mass agents [1]. In [5], a new rearrangement strategy is proposed for producing desired attractive and repulsive forces for CMCA of mobile robots. In [1], [8], [16], the graph theory was introduced to generate control protocols that maintain CMCA of multi-agent systems whose models are double integrators. In this paper, we study agents with nonholonomic dynamics and boundedness constraints. Specifically, the coordination function is redesigned to ensure that the induced attractive and repulsive forces are bounded, and hence can be included in the bounded velocity control. With the help of Barbalat's lemma and graph theory, the basin of attraction for the flocking convergence is determined by the maximal value of the coordination function.

In the context of the current paper, we are concerned with the problem of leaderless flocking for a group of nonholonomic vehicles, which invokes graph theory as in the case of nearest neighbor communication [1], [8]. Also, we are interested in the plain velocity consensus without specifying the desired heading direction. These simplifying conditions allow us to focus on the introduction of our bounded control design.

II. PROBLEM FORMULATION

Notations: $\mathbb{R}$ and $\mathbb{R}^+$ are the sets of real numbers and nonnegative real numbers, respectively; for $q = [q_1, \ldots, q_n]^T$, $\nabla_q = [\partial/\partial q_1, \ldots, \partial/\partial q_n]^T$ is the del operator [20]; for two vectors a and b, $a \cdot b$ is their scalar product; $(a_1, \ldots, a_n)$ is $[a_1^T, \ldots, a_n^T]^T$; $|\cdot|$ is the absolute value of scalars; and $\|\cdot\|$ is the Euclidean norm of vectors.

Consider a collective system of N identical autonomous

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mobile robots whose respective equations of motion are [18]

$$\dot{q}_i = v_i e(\theta_i)$$
$$\dot{\theta}_i = w_i$$
$$\dot{v}_i = u_i$$
$$\dot{w}_i = \tau_i \quad (1)$$

where $i = 1,...,N$, $q_i = [x_i, y_i]^T \in \mathbb{R}^2$, and $\theta_i \in \mathbb{R}$ are respectively the position and the heading angle of the i-th robot in the inertial frame Oxy; $v_i \in \mathbb{R}$ is the linear speed, and $e(\theta_i)$ is the unit vector $[\cos\theta_i, \sin\theta_i]^T$; $w_i \in \mathbb{R}$ is the angular speed, and $u_i, \tau_i \in \mathbb{R}$ are control inputs.

Let r_0, R_0 be positive constants, $r_0 < R_0$. Our flocking control problem for (1) is to obtain the controls u_i, τ_i as bounded functions of the collective state $(q_1,\ldots,q_N,\theta_1,\ldots,\theta_N,v_1,\ldots,v_N,w_1,\ldots,w_N)$ in a distributed fashion such that the following multiple goals are achieved

G1) *Velocity consensus:*

$$\lim_{t\to\infty}(\dot{q}_i(t) - \dot{q}_j(t)) = 0, \forall i, j = 1,\ldots,N \quad (2)$$

G2) *Collision avoidance:* $r_{ij}(t) = \|q_i(t) - q_j(t)\| \geq r_0, \forall t \geq 0, \forall i \neq j$

G3) *Cohesion maintenance:* $r_{ij}(t) \leq R_0, \forall t \geq 0, \forall i \neq j$.

For disambiguation, we have the following definition.

Definition 2.1: A control $\dot{\zeta} = g(\zeta, y), u = c(\zeta, y), (\zeta, y) \in \mathbb{R}^d \times \mathbb{R}^m$ of a system $\dot{x} = f(x,u), y = h(x,u)$ is said to be bounded if there is a finite constant $M > 0$ such that $\|c(\zeta, y)\| \leq M, \forall (\zeta, y) \in \mathbb{R}^d \times \mathbb{R}^m$.

To achieve the goals G2) and G3), we consider the coordination function $U : \mathbb{R}^+ \to \mathbb{R}^+$ which satisfies the following properties:

P1) there is a constant $U_M > 0$ such that

$$0 \leq U(r) \leq U_M, \forall r \in \mathbb{R} \quad (3)$$

P2) $U(r)$ is continuously differentiable on $[r_0, R_0]$;

P3) $\lim_{r\to r_0^+} U(r) = U_M$; and

P4) $\lim_{r\to R_0^-} U(r) = U_M$.

Since we are maintaining $r_{ij}(t) \in [r_0, R_0]$, without loss of generality, we assume that $U(0) = 0$ and hence $U(r)$ is well defined for $r_{ii} = 0$.

We are interested in the function U with the dead zone $[a, A]$ since evenly distribution of agents may not be achievable by a common coordination function U. Accordingly, we use the zone $[a, A]$ for free alignment.

For bounded control, we shall use the linear saturation functions σ_1, σ_2 and σ_3, which are continuous and nondecreasing functions and satisfy, for given positive constants $L_i \leq M_i, i = 1,2,3$

i) $\sigma_i(-s) = -\sigma_i(s)$ for all s;

ii) $\sigma_i(s) = s$ for $s \leq L_i$; and

iii) $|\sigma_i(s)| \leq M_i, \forall s \in \mathbb{R}$.

For bounded backstepping, we shall use the scaling function Ω [21], which is a real-valued and continuously differentiable and satisfies, for a positive constant B,

Ω1) $\Omega(s) = s, \forall s \in [-2B, 2B]$; and

Ω2) $\Omega'(s) \geq 1, \forall s$.

Similarly to other works on distributed for multi-agent systems [1], [8], [16], we will employ graph theory to address our problem. A digraph associated with (1) is defined as $\mathscr{G}(t) = (\mathscr{V}, \mathscr{E}(t))$ where $\mathscr{V} = 1,\ldots,N$ and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$. The set $\mathscr{V}$ is called the node set of $\mathscr{G}(t)$ and the set $\mathscr{E}(t)$ is defined as the edge set of $\mathscr{G}(t)$. In addition, $\mathscr{N}_i(t)$ denotes the neighbor set of the node i for $i = 1,\ldots,N$.

As in [16], the description of the edge $\mathscr{E}(t)$ is presented as follows.

Given any $R > 0$, $\varepsilon_2 \in (0,R)$, and $\varepsilon_1 \in (0, R - \varepsilon_2)$, for any $t \geq 0$, $\mathscr{E}(t) = \{(i,j) | i,j \in \mathscr{V}\}$ is defined such that

1) $\mathscr{E}(0) = \{(i,j) | \varepsilon_1 < \|q_i(0) - q_j(0)\| < (R - \varepsilon_2)\}$;
2) if $\|q_i(0) - q_j(0)\| \geq R$, then $(i,j) \notin \mathscr{E}(t)$;
3) for $i = 1,\ldots,N$, $j = 1,\ldots,N$, if $(i,j) \notin \mathscr{E}(t^-)$ and $\|q_i(t) - q_j(t)\| < R - \varepsilon_2$, then $(i,j) \in \mathscr{E}(t)$;
4) for $i = 1,\ldots,N$, $j = 1,\ldots,N$, if $(i,j) \in \mathscr{E}(t^-)$ and $\|q_i(t) - q_j(t)\| < R$, then $(i,j) \in \mathscr{E}(t)$.

The following results will be employed for the main results.

Lemma 2.1: Let $\sigma : \mathbb{R} \to \mathbb{R}$ be a function satisfying $\sigma(-s) = -\sigma(s), \forall s \in \mathbb{R}$. Then, for all a_i, b_i, it holds true that

$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t)}(a_i - a_j)\sigma(b_i - b_j) = \sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t)} a_i \sigma(b_i - b_j). \quad (4)$$

Proof: See Appendix. ■

Lemma 2.2: The linear saturation functions $\sigma_i, i = 1,2,3$ satisfy

$$(\sigma_i(\theta_1) - \sigma_i(\theta_2))\sigma_i(\theta_1 - \theta_2) \geq 0, \forall \theta_1, \theta_2. \quad (5)$$

Proof: See Appendix. ■

III. MAIN RESULTS

Our design strategy is to design u_i to achieve consensus on v_i, and design τ_i to achieve consensus on θ_i. As τ_i is not the direct input of θ_i dynamics, the backstepping procedure in [21] will be employed.

Since $U(r_{ij}) = U(\|q_i - q_j\|)$, in the following, we shall consider U as the symmetric function of q_i and q_j, and we write $U(q_i, q_j)$ with the understanding that $U(q_i, q_j) = U(q_j, q_i)$. Our design is Lyapunov-based. Particularly, we shall construct a positive definite function V and solve for the protocol u_i and τ_i such that the time derivative of V is a negative definite function. The graph theory will be exploited to show the connectivity preservation for our multi-agent system. Then, we apply the LaSalle's invariance principle [22] to conclude the desired consensuses.

The initial state of the collective system of agents (1) is chosen such that the graph $\mathscr{G}(0)$ is connected. The parameters of the graph $\mathscr{G}(0)$ are chosen as follows

$$R = R_0, \quad (6)$$
$$r_0 \leq \varepsilon_1 < a, \quad (7)$$
$$0 < \varepsilon_2 \leq R_0 - a. \quad (8)$$

A. Speed consensus and connectivity perservation

Consider the energy function for system (1)

$$V_1 = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t)} U(q_i,q_j) + \frac{1}{2}\sum_{i=1}^{N} v_i^2. \quad (9)$$

Assume that $U(r)$ is designed such that

$$U(R_0) = U_M \geq V_{1max}, \quad (10)$$

where

$$V_{1max} \triangleq \frac{1}{2}\sum_{i=1}^{N} v^2(0) + \frac{N(N-1)}{2}U(R_0-\varepsilon_2). \quad (11)$$

Let m_0 be the number of the links of the initial graph. The simplest connected graph of N agents is a tree whose number of links is $n-1$. Hence, $m_0 \geq n-1$. Therefore,

$$V_1(0) \leq V_{1max} - \frac{(N-1)(N-2)}{2}U(R_0-\varepsilon_2). \quad (12)$$

Note that $U(q_i,q_j)$ is a symmetric function of q_i and q_j. We compute the derivative of V_1 with respect to (1)

$$\begin{aligned}\dot{V}_1 &= \sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t)} v_i\nabla_{q_i}U(q_i,q_j)\cdot\dot{q}_i + \sum_{i=1}^{N} v_i u_i \\ &= \sum_{i=1}^{N} v_i\left(\sum_{j\in\mathscr{N}_i(t)}\nabla_{q_i}U(q_i,q_j)\cdot e(\theta_i) + u_i\right). \quad (13)\end{aligned}$$

From (13), a design for speed consensus protocol is chosen as

$$u_i = -\sum_{j\in\mathscr{N}_i(t)}\nabla_{q_i}U(q_i,q_j)\cdot e(\theta_i) - \sum_{j\in\mathscr{N}_i(t)}\sigma_1(v_i-v_j) \quad (14)$$

where σ_1 is the linear saturation function introduced in Section II.

Substituting (14) into (13), we obtain

$$\dot{V}_1 = -\sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t)} v_i\sigma_1(v_i-v_j). \quad (15)$$

We have the following speed consensus theorem.

Theorem 3.1: Suppose that the collective system (1) subject to the protocol (14) is initiated such that $V_1(0) < V_{1max}$. Then, the following properties hold true:

i) $\mathscr{G}(t)$ is connected for all $t \geq 0$ and there exists t_k such that for $t \geq t_k$, $\mathscr{G}(t) = \mathscr{G}(t)$
ii) $\lim_{t\to\infty}(v_i(t) - v_j(t)) = 0$.

Proof: See Appendix. ■

By Theorem 3.1, the design (14) achieves speed consensus and the goals G2) and G3). We have the following subsection designing τ_i for orientation consensus completing the goal G1).

B. Orientation Consensus

Since the dynamics of θ_i is a double integrator, we shall develop a bounded backstepping approach which is motivated by the result [21] for single nonlinear systems.

Consider the function

$$V_2 = \sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t_k)}\int_0^{\theta_i-\theta_j}\sigma_2(s)ds. \quad (16)$$

It is seen that $V_2 = 0$ since the graph is undirected.

The derivative of V_2 is

$$\begin{aligned}\dot{V}_2 &= \sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t_k)}\sigma_2(\theta_i-\theta_j)(w_i-w_j) \quad (17)\\ &= -\sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t_k)}\sigma_2(\tilde{\theta}_{ij})(\sigma_2(\theta_i)-\sigma_2(\theta_j)) \\ &+\sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t_k)}\sigma_2(\tilde{\theta}_{ij})(w_i+\sigma_2(\theta_i)-(w_j+(\sigma_2(\theta_j)) \quad (18)\end{aligned}$$

where $\tilde{\theta}_{ij} = \theta_i - \theta_j$. Using Lemma 2.1, we have

$$\begin{aligned}\dot{V}_2 \;=\; &-\sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t_k)}\sigma_2(\tilde{\theta}_{ij})(\sigma_2(\theta_i)-\sigma_2(\theta_j)) \\ &+2\sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t_k)}\sigma_2(\tilde{\theta}_{ij})(w_i+\sigma_2(\theta_i)). \quad (19)\end{aligned}$$

To this end, we augment V_2 to obtain the function

$$V_3 = V_2 + \frac{1}{2}\sum_{i=1}^{N}\left(\Omega(w_i)+\sigma_2(\theta_i)\right)^2 \quad (20)$$

where Ω is the scaling function introduced in Section II.

To design the control τ_i bounded, let us define the variables

$$\begin{aligned}\xi_i &= w_i + \sigma_2(\theta_i), \\ \Omega_i &= \Omega(w_i) + \sigma_2(\theta_i). \quad (21)\end{aligned}$$

For brevity, let

$$\tilde{\theta}_{ij} = \theta_i - \theta_j, .$$

We have

$$\dot{\Omega}_i = \Omega'(w_i)\tau_i + \sigma_2'(\theta_i)w_i. \quad (22)$$

From (20) and (19), we have

$$\begin{aligned}\dot{V}_3 &= -\sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t)}(\sigma_2(\theta_i)-\sigma_2(\theta_j))\sigma_2(\tilde{\theta}_{ij}) + 2\sigma_2(\tilde{\theta}_{ij})\xi_i \\ &+\sum_{i=1}^{N}\Omega_i\left(\Omega'(w_i)\tau_i+\sigma_2'(\theta_i)w_i\right) \\ &= -\sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i(t)}(\sigma_2(\theta_i)-\sigma_2(\theta_j))\sigma_2(\tilde{\theta}_{ij}) \\ &+\sum_{i=1}^{N}\xi_i\left(\frac{\Omega_i}{\xi_i}\Omega'(w_i)\tau_i + \frac{\Omega_i}{\xi_i}\sigma_2'(\theta_i)w_i + 2\sum_{j\in\mathscr{N}_i(t)}\sigma_2(\tilde{\theta}_{ij})\right). \quad (23)\end{aligned}$$

Using (23), we have the following design for bounded τ_i

$$\tau_i = -\frac{\xi_i}{\Omega_i}\frac{1}{\Omega_i'}\left(\sum_{j\in\mathcal{N}_i(t)}\sigma_3(\xi_i-\xi_j)+2\sigma_2(\tilde{\theta}_{ij})\right) - \frac{w_i}{\Omega_i'}\frac{1}{N}\sum_{j=1}^{N}\sigma_2'(\theta_i). \quad (24)$$

We now verify that τ_i is well-defined. Indeed, by property Ω1) of the function Ω introduced in Section II, we have

$$\Omega(w_i) = w_i, \text{ for } w_i \in [-2B, 2B]. \quad (25)$$

Accordingly, in view of (21), for $w_i \in [-2B, 2B]$, we have

$$\frac{\xi_i}{\Omega_i} = 1. \quad (26)$$

For $w_i \notin [-2B, 2B]$, we have $|\Omega(w_i)| > 2B$. Accordingly, choosing the saturating value M_2 of the function σ_2 satisfying

$$M_2 < 2B \quad (27)$$

we have

$$|\sigma_2(\theta_i)| < 2B \quad (28)$$

which implies that $|\Omega_i| > 0$ for $w_i \notin [-2B, 2B]$, and hence ξ_i/Ω_i is well defined.

Furthermore, as $|\Omega(s)| \geq |s|, \forall s$, the design (28) guarantees that $|\xi_i| \leq |\Omega_i|$ for $w_i \notin [-2B, 2B]$. This and (26) indicate that

$$\left|\frac{\xi_i}{\Omega_i}\right| \leq 1, \forall i. \quad (29)$$

Note that $\Omega(s)$ can be chosen such that w_i/Ω_i' is bounded $\forall s$; see [21] for more details. This and (29) indicate that the steering law (24) is well-defined.

Substituting (24) into (23) and using (19), we arrive at

$$\dot{V}_3 = -\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)}(\sigma_2(\theta_i)-\sigma_2(\theta_j))\sigma_2(\theta_i-\theta_j) - \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)}\xi_i\sigma_3(\xi_i-\xi_j). \quad (30)$$

Applying Lemma 2.1 to the last term of (30), we obtain

$$\dot{V}_3 = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)}(\sigma_2(\theta_i)-\sigma_2(\theta_j))\sigma_2(\theta_i-\theta_j) - \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)}(\xi_i-\xi_j)\sigma_3(\xi_i-\xi_j). \quad (31)$$

We have the following orientation consensus theorem.

Theorem 3.2: Suppose that the collective system (1) is subject to the protocol (24). Then, all the mobile robots eventually reach consensus on the heading angles θ_i, i.e.,

$$\lim_{t\to\infty}(\theta_i(t)-\theta_j(t)) = 0, \forall i, j. \quad (32)$$

Proof: See Appendix. ■

Combining Theorems 3.1 and 3.2, we have the following bounded flocking theorem.

Theorem 3.3: Suppose that the collective system (1) is subject to the bounded protocols (14) and (24). Suppose further that the initial configuration of the collective system (1) is such that $\mathcal{N}(0)$ is connected, and the design parameters satisfy (27). Then, all the multiple flocking goals of velocity consensus, cohesion maintenance, and collision avoidance are achieved.

Proof: The proof is straightforward from the results of Theorems 3.1 and 3.2. ■

IV. SIMULATION

We run simulation for a multi-agent system of 10 mobile robots of the model (1). A bump function is used to generate the smooth coordination function U. As the control (14) invokes the gradient forces $\nabla_{q_i}U$, we designed the coordination function in the form

$$U(r) = \int_0^r \varphi(s)ds \quad (33)$$

where φ is a compact support function given by

$$\varphi(s) = \begin{cases} p_1\exp\left(\frac{-(s-s_0)^2}{((a-r_0)/2)^2-(s-s_0)^2}\right) & \text{if } s\in(r_0,a) \\ p_2\exp\left(\frac{-(s-s_1)^2}{((R_0-A)/2)^2-(s-s_1)^2}\right) & \text{if } s\in(A,R_0) \\ 0 & \text{otherwise} \end{cases}$$

where $s_0 = \frac{r_0+a}{2}$, $s_1 = \frac{R_0+A}{2}$, and p_1, p_2, a, A, r_0 and R_0 are design parameters.

The parameters of the coordinate function are $r_0 = 1$, $a = 3$, $A = 6$, $R_0 = 9$, and $U_M = 20$. The parameter for the scaling function Φ is $B = 30$. The initial positions of 10 mobile robots are $X_0 = [-2.5; 0.5; -3; 0; -4; -5; -5.5; -6; -6.5; -7]$ and $Y_0 = [-9.5; -8; -6; -4; -2; 0; 2; 4; 6; 8]$ where X_0 and Y_0 are respectively the vectors of x and y coordinates of the robots.

We obtained the simulation results shown in Fig. 1–Fig. 5. The flocking behavior is shown in Fig. 1, where no collision occurred. It is shown that consensuses on orientation and speed of the mobile robots have been obtained in Fig. 2 and Fig. 3, respectively. The trajectories of mobile robots are depicted in Fig. 1. The control signals are shown to be bounded in Fig. 4 and Fig. 5.

V. CONCLUSIONS

Bounded protocol for decentralized flocking control of mobile robots has been constructed by a systematic design, in which limited communication is introduced. Theoretical results have proved that the proposed scheme helps a collective system of mobile robots achieve all the multiple objectives of the flocking control: velocity consensus, cohesion maintenance, and collision avoidance. The numerical results have shown the efficiency of the proposed protocol design. Future work will focus on the flocking control of mobile robots in noisy environments [23], [24].

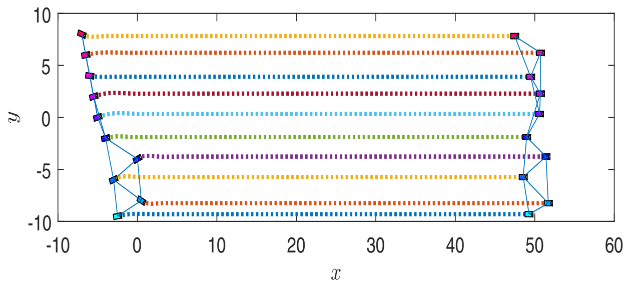

Fig. 1. Distributed flocking of 10 mobile robots.

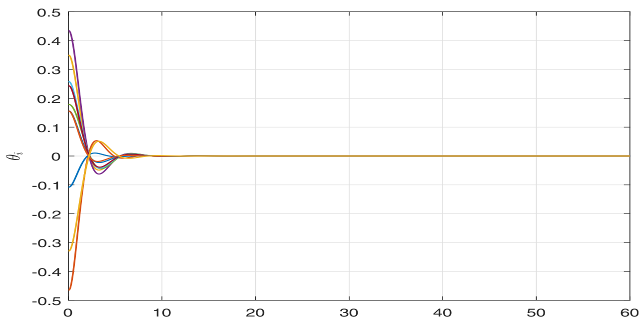

Fig. 2. Orientation consensus.

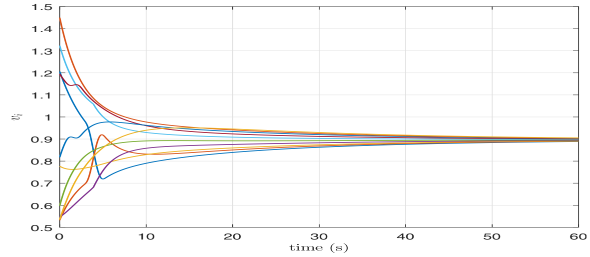

Fig. 3. Speed consensus.

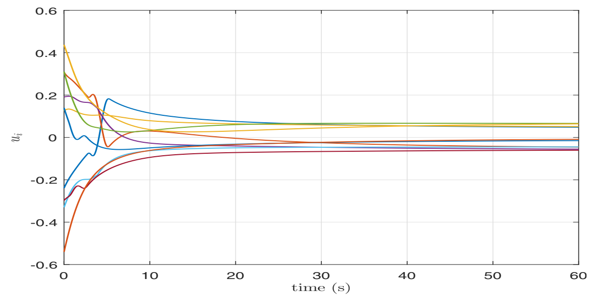

Fig. 4. Speed control.

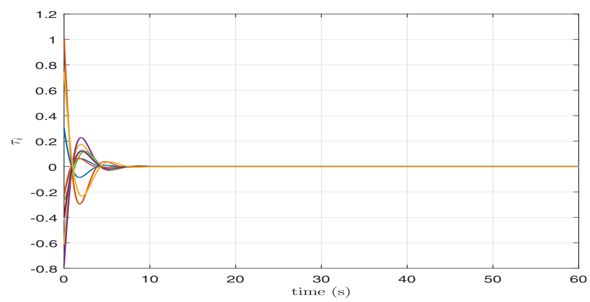

Fig. 5. Steering control.

APPENDIX

A. Proof of Lemma 2.1

Proof: Since $\sigma(-s) = -\sigma(s)$ and $\mathscr{G}(t)$ is an undirected graph, we have

$$\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} a_j\sigma(b_i-b_j) = -\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} a_j\sigma(b_j-b_i)$$
$$= -\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} a_i\sigma(b_i-b_j). \quad (34)$$

Hence,

$$\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} (a_i-a_j)\sigma(b_i-b_j)$$
$$= \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} a_i\sigma(b_i-b_j) - \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} a_j\sigma(b_i-b_j)$$
$$= 2\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} a_i\sigma(b_i-b_j) \quad (35)$$

which implies (4). ■

B. Proof of Lemma 2.2

Proof: Without loss of generality, suppose that $\theta_1 \geq \theta_2$. Since σ_i are nondecreasing functions, this implies that

$$\sigma_i(\theta_1) - \sigma_i(\theta_2) \geq 0. \quad (36)$$

Furthermore, as $\sigma_i(0) = 0$, $\theta_1 \geq \theta_2$ and the nondecreasing property of σ_i imply that

$$\sigma_i(\theta_1 - \theta_2) \geq 0. \quad (37)$$

Multiplying (36) and (37) side-by-side, we obtain (5). ■

C. Proof of Theorem 3.1

Proof:

Assume that $\mathscr{G}(t)$ switches at time t_k $(k = 1, 2, \dots)$. Hence, $G(t) = G(0)$ for all $t \in [0, t_1)$. In other words,

$$\mathscr{G}(t) = G(0), \ t \in [0, t_1)$$
$$\mathscr{G}(t_1) \neq G(0). \quad (38)$$

We show that $G(0) \subset G(t_1)$. Under the control law (14), the time derivative of V_1 in $[0, t_1)$ is

$$\dot{V}_1 = -\sum_{i=1}^{N} v_i \sum_{j\in\mathcal{N}_i(t)} \sigma_1(v_i - v_j). \quad (39)$$

According to Lemma 2.1, we have

$$\dot{V}_1 = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} (v_i - v_j)\sigma_1(v_i - v_j). \quad (40)$$

Since $\sigma_1(s)$ is an odd function, $(v_i - v_j)\sigma_1(v_i - v_j) \geq 0$. Hence, $\dot{V}_1 \leq 0$, which implies that

$$V_1(t) \leq V_1(0) \leq V_{1max} < U_M \text{ for } [0, t_1). \quad (41)$$

From the definition of $U(r)$, $U(R_0) > V_{1max} \geq V_1(0)$. Hence for any $(i,j) \in \mathscr{G}(t)$ for $t \in [0,t_1)$

$$U(q_i,q_j) \leq V_1(t) < U_M = U(r_0) = U(R_0). \quad (42)$$

By the continuity of $U(r)$, (42) shows that $r_0 < \|q_i - q_j\| < R_0$. This implies that no existing links are deleted at time t_1 and collision avoidance is achieved. Hence, new links must be added to the current graph at the switching time t_1. Assume that there are m_1 new links being added to the network at time t_1. On one hand, the number of the current links before switching is $m_0 \geq N-1$. On the other hand, the complete graph possesses $\frac{N(N-1)}{2}$ edges. As a result, $m_1 \leq \frac{N(N-1)}{2} - (N-1) = \frac{(N-1)(N-2)}{2}$. Then,

$$V_1(t_1) = V_1(t_1^-) + m_1 U(R_0 - \varepsilon_2). \quad (43)$$

According to (12),

$$V_1(t_1^-) \leq V_1(0) \leq V_{1max} - \frac{(N-1)(N-2)}{2} U(R_0 - \varepsilon_2). \quad (44)$$

Hence,

$$V_1(t_1) \leq V_{1max}. \quad (45)$$

By induction, for $t \in [t_{k-1}, t_k)$,

$$\dot{V}_1 = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j \in \mathscr{N}_i(t)} (v_i - v_j) \sigma_1 (v_i - v_j), \quad (46)$$

and $V_1(t) \leq V_1(t_{k-1}) \leq V_{1max}$. This implies no edges are lost at time t_k and $V_1(t_k) \leq V_{1max}$. Hence, the size of the set of the links of $\mathscr{G}(t)$ forms an increasing sequence, bounded above by $\frac{N(N-1)}{2}$, which is the number of the links of a complete graph. Hence. there exists a finite integer $k > 0$ such that

$$\mathscr{G}(t) = \mathscr{G}(t_k), \ t \in [t_k, \infty). \quad (47)$$

Therefore, for $t \geq t_k$, we have

$$\dot{V}_1 = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j \in \mathscr{N}_i(t_k)} (v_i - v_j) \sigma_1 (v_i - v_j) \leq 0. \quad (48)$$

Now we are in a position to show that the linear velocities of all agents converge to the same value. Since $0 \leq V_1(t) \leq V_{1max}$ and $\dot{V}_1 \leq 0$, by Barbalat's lemma, $\lim_{t\to\infty} \dot{V}_1(t) = 0$. Since the graph $\mathscr{G}(t)$ is connected for all t and $s\sigma_1(s) \geq 0$ for all s, from (48),

$$\lim_{t\to\infty} (v_i - v_j) = 0, \text{ for all } i,j = 1,2,\ldots,N. \quad (49)$$

■

D. Proof of Theorem 3.2

Proof: By Lemma 2.2, the right-hand-side of (31) is negative definite. Hence, $V_3(t)$ is nonincreasing in each interval $[t_{k-1}, t_k)$. As pointed in the proof of Theorem 3.1, (47) holds. Hence, for $t \geq t_k$, a standard application of Barbalat's lemma [22] to (31) indicates that the right-hand-side of (31) converges to zero. Furthermore, from Theorem 3.1, the graph $G(t)$ is connected for all t, which verifies the conclusion of the theorem. ■

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