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Distributed Formation Control for Autonomous Robots Following Desired Shapes in Noisy Environment

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***Abstract*— In this paper, we propose a novel and distributed formation control method for autonomous robots to follow the desired formation while tracking a moving target under influence of the dynamic and noisy environments. In our approach, the desired formations, which include the virtual nodes arranged into specific shapes, are first generated. Then, autonomous robots are controlled by the proposed artificial force fields in order to converge to these virtual nodes without collisions. The stability analysis based on the Lyapunov approach is given. Moreover, a new combination of rotational force field and repulsive force field in designing an obstacle avoidance controller allows the robot to avoid and escape the convex and non convex obstacle shapes. The V-shape and circular shape formations with their advantages are utilized to test the effectiveness of the proposed method.**

***Keywords*: Multi-robot systems, Mobile sensor networks, Formation control, collision avoidance**

I. Introduction

In recent years, multi-agent systems have widely been researched in many areas, such as physic, biology, cybernetics, and automatic control over the world. Formation control is one of the necessary and important problems in the research field on multi-agent systems. The formation control of autonomous robots, such as unmanned aerial vehicles [8], underwater vehicles [9], mobile sensor networks [14, 15], etc., has potential applications in search and rescue missions, forest fire detection and surveillance, etc.

There are several research directions on multi-robot systems, but the main aim is that the robot team has to work together in order to achieve the desired tasks, such as tracking and observing a moving target. Formation control of autonomous robots is inspired from natural behavior of fish schooling, bird flocking or ant swarming and guaranteed that the members in the formation have to move together under the velocity matching and collision avoidance. There are several methods to generate and control the formation of a swarm of mobile robots. Artificial potential field is known as a positive tool in order to control the coordination and the motion of a swarm towards the target position, see [1-7]. The success of the formation control method based on the random connections among neighboring members in a swarm as an α-lattice configuration has been published in some literature, such as [14-18]. In this method, the neighboring robots are linked to each other by the attractive/repulsive force fields among them to create a robust formation without collisions. On the other hand, the formation control based on dynamic framework was introduced [10, 11] in which the robots are able to adjust their formation by rotating and scaling during their movement. In another approach, robots are controlled to achieve given positions in the desired shape [12, 13]. Although artificial potential field is known as a positive method for path planning of mobile robots, but in several cases of local minimum problems this approach is still limited. Namely, when the attractive force of the target and the repulsive force of the obstacles are equal and collinear but opposite direction, the total force on the robot is equal to zero. Hence, this causes the robot motion stopped. Moreover, in complex environments with convex and concave obstacle shapes, such as U-shaped obstacles or long walls, etc., the application of the traditional potential field method is very difficult. Robots can be trapped in these obstacles before reaching the target, see [1-3].

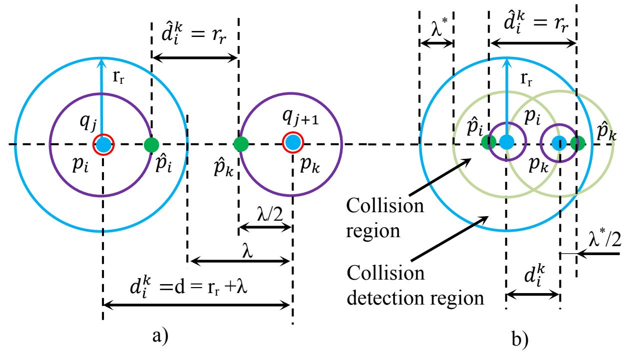

Fig.1. Description of the noise's boundary guarantees that there is no effect to each other between two active neighboring robots i and k (a), and there is no collision between robots i and k (b). Where d_i^k and $\hat{d}_i^k$ are the actual distance and the estimate distance between robots, respectively.

In summary, the main contributions of this paper are as follows. The distributed formation control algorithms are designed in order to control multiple robots to converge to the desired positions under the influence of the noisy environments, see Fig.1. These control algorithms guarantee that the stability of the formation is maintained, and there are no collisions among robots while tracking a moving target. Furthermore, the obstacle avoidance control algorithm is built based on the combination of the rotational force field and the repulsive force field surrounding the obstacles in order to drive robots to escape these obstacles without collisions. The stability analysis of the proposed control algorithms based on Lyapunov approach is given. In case study, V-shape and circular shape with their advantages are utilized

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as the desired formations to test the effectiveness of the proposed control algorithms.

The remaining sections of this paper are organized as follows: The problem formulation is presented in the section II. Section III presents the formation control algorithms. Simulation results are discussed in section IV. Finally, section V concludes the paper and proposes the future research topics.

II. Problem Formulation

In this section, we consider a swarm of N robots and their mission is to track a moving target in two-dimensional space. Let $p_i=(x_i,y_i)^T$, $v_i=(v_{ix},v_{iy})^T$ be the position, velocity vectors of the robot i $(i=1,2,3,...,N)$, respectively. The dynamic model of the robot i is described as follows:

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= u_i, \qquad i=1,..,N. \end{aligned} \tag{1}$$

The formation of autonomous robots must satisfy the following conditions: All robots will converge to the desired positions in the desired formation. While tracking a moving target in noisy environment, the stability of the formation must be maintained, and there are no collisions among members. Additionally, robots must also automatically escape the obstacles in order to continue to track the moving target with their swarm. Hence, in order to solve these problems we propose the control input u_i for each robot as follows:

$$u_i = \begin{cases} u_i^t + u_i^o, & \text{if robot } i \text{ is leader } l,\ i=1,..,N \\ u_i^j + u_i^o + u_i^k, & \text{otherwise.} \end{cases} \tag{2}$$

Where, the first controller u_i^j is used to control the formation connection. The second controller u_i^o is used to drive robots to avoid obstacles. The controller u_i^k is added to help robots avoid collision during their movement. Using the tracking controller u_i^t the leader can easily drive its swarm towards the target. Now, in order to design these controllers, firstly, we have some definitions and remarks as follows:

Definition 1. Robot i $(i=1,2,3,...,N)$ is called an active robot at time t if the distance from it to the virtual node j $(j=1,2,3,...,N)$ is smaller than the radius of the active circle surrounding each virtual node (d_i^j < r_a, $r_a=d/2-\lambda_a$, λ_a is a positive factor), see Fig.2. Otherwise, it is a free robot.

Definition 2. Virtual node j $(j=1,2,..,N;\ q_j=(x_j, y_j)^T;\ v_j=(v_{jx}, v_{jy})^T)$ of the desired formation is active if there is a robot i $(i=1,2,..,N)$ in the active circle of this virtual node, see Fig.2. In contrast, it is free.

Definition 3. Desired position for each robot i in the desired formation is a virtual node j at which $\lim_{t\to\infty}(p_i(t)-q_j(t)) = 0$, and the virtual node $(j\text{-}1)$ is also active.

Definition 4. The desired V-shape formation is a formation that is linked by two line formations. These line formations own a leader together and are connected by a formation angle φ. In the line formations, the virtual nodes are equidistant each other.

Definition 5. The desired circular shape formation is the locus of all virtual nodes that are equidistant each other and equidistant from the target.

Remark 1. Consider a desired formation (V-shape or circular shape formation) of N virtual nodes as shown in Fig.2. Each robot must find a desired position in this desired formation. Firstly, each free robot i will pursue a closest free virtual node j in order to be active at this virtual node. If the position of an active robot at the active node j is still not desired position (for example robot k in Fig.2 with $k=1,2,..,N,\ k\neq l$), then this active robot will automatically move into the virtual nodes $(j\text{-}1)$ until it achieves a desired position.

Remark 2. The motion of the formation depends on the relative position between the leader and the target. At initial time, a robot, which is closest to the target, is chosen as a leader to lead its formation towards the target. During movement, if the actual leader encounters any risk, such as it is broken or hindered by the environment, then a new leader is replaced. This new leader will reorganize the formation and continue to lead the new formation to track the target.

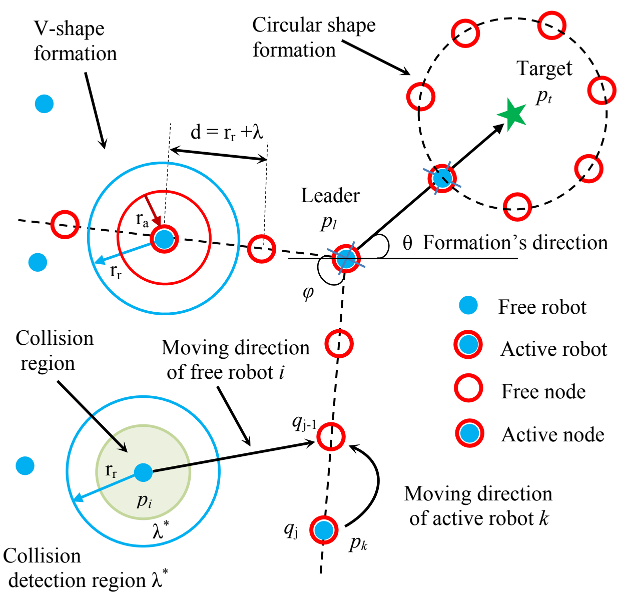

Fig.2. Description of the formation control method following the V-shape desired structure.

III. Control Algorithms

This section presents control algorithms that guarantee the conditions as presented in section II.

A. Distributed formation control

Firstly, surrounding the virtual nodes j $(j=1,2,...N)$, the attractive force fields are created to drive the free robots towards the desired positions. Then, these free robots will occupy these desired positions, and become active robots. The tracking task is to make the distance $d_i^j = \|p_i - q_j\|$ approaching to zero as fast as possible. This means that $\lim_{t\to\infty}(p_i(t)-q_j(t)) = 0$ and $\lim_{t\to\infty}(v_i(t)-v_j(t)) = 0$. Based on this analysis, the formation control law for formation connection is proposed in Algorithm 1.

In Algorithm 1, $k_{ip1}^j, k_{ip2}^j, k_{ip3}^j, k_{iv}^j$, $(p_i - q_j)$ and $(v_i - v_j)$ are the positive gain factors, the relative position vector, the relative velocity vector between the robot i and the virtual node j, respectively. In this algorithm, we use two potential fields $f_{1i}^j = -k_{ip3}^j(p_i - q_j)/\|p_i - q_j\|$ and $f_{2i}^j = -k_{ip1}^j(p_i - q_j)$ as the artificial attractive forces. The constant potential field f_{1i}^{j} is used to drive the free robots towards the desired formation, while the linear potential field f_{2i}^{j} is used to control the active robots to approach to the virtual nodes. Additionally, the component $-k_{iv}^{j}(v_i - v_j)$ is also utilized as the damping term. Therefore, using the Algorithm 1, robots can quickly approach the desired positions at the virtual nodes of the desired formation.

Algorithm 1: Reaching the desired position at the virtual nodes in the desired formation

Consider: a robot *i* and virtual nodes *j* (*i,j=1,..,N, i≠l*).
Determine the shortest distance from p_i to all the virtual nodes q_j and the scalar factor c_i^j given as

$$d_i^{j\,\mathrm{m1}} = \min\left\{d_i^j = \left\|p_i - q_j\right\|,\ j = 1,..,N\right\}, c_i^j = \begin{cases}1 & \text{if } q_j \text{ is active}\\ 0 & \text{if } q_j \text{ is free.}\end{cases}$$

if $d_i^{j\,\mathrm{m1}} \le r_a$ & $c_i^{j\,\mathrm{m1}-1} = 1$ **then**

$$u_i^j = -k_{ip1}^j (p_i - q_{jm1}) - k_{iv}^j (v_i - v_{jm1}) + \dot{v}_{jm1}$$

else if $d_i^{j\,\mathrm{m1}} \le r_a$ & $c_i^{j\,\mathrm{m1}-1} = 0$ **then**

$$u_i^j = -k_{ip2}^j (p_i - q_{jm1-1}) - k_{iv}^j (v_i - v_{jm1-1}) + \dot{v}_{jm1-1}$$

else if $d_i^{j\,\mathrm{m1}} > r_a$ **then**

if $c_i^{j\,\mathrm{m1}} = 0$ **then**

$$u_i^j = -k_{ip3}^j (p_i - q_{jm1}) / \left\|p_i - q_{jm1}\right\| - k_{iv}^j (v_i - v_{jm1}) + \dot{v}_{jm1}$$

else

Determine the shortest distance from p_i to the free virtual nodes q_j in the desired formation as

$$d_i^{jm2} = \min\left\{d_i^j = \left\|p_i - q_j\right\|, c_i^j = 0,\ j = 1,..,N,\ j \ne jm1\right\}$$

$$u_i^j = -k_{ip3}^j (p_i - q_{jm2}) / \left\|p_i - q_{jm2}\right\| - k_{iv}^j (v_i - v_{jm2}) + \dot{v}_{jm2}$$

end

end

To consider the stability of the formation under the influence of noises, which cause the position errors between the robot *i* and the virtual node *j*, we assume that the estimates of the position and the velocity of the robot *i* are $\hat{p}_i = p_i + z_{pi}$ and $\hat{v}_i = v_i + z_{vi}$, where z_{pi} and z_{vi} are the position and velocity measurement errors of the robot *i*, respectively. Similarly, the estimates of the position and the velocity of the virtual node *j* are also defined as: $\hat{q}_j = q_j + z_{qj}$ and $\hat{v}_j = v_j + z_{vj}$, where z_{qj} and z_{vj} are the position and velocity noises of the node *j*, respectively. Now, we propose a new control law for the robot *i* at the active node *j* in noisy environment as follows:

$$u_i^j = -\hat{k}_{ip1}^j (\hat{p}_i - \hat{q}_j) - \hat{k}_{iv}^j (\hat{v}_i - \hat{v}_j) + \dot{v}_j - \dot{z}_{vij}. \qquad (3)$$

Where, $\hat{k}_{ip1}^j = k_{ipd}^j + \varepsilon_2 \left\|\hat{v}_j\right\|$ and $\hat{k}_{iv}^j$ are the positive factors. Let $\hat{x}_1 = \hat{p}_i - \hat{q}_j = p_i - q_j + z_{pij}$ and $\hat{x}_2 = \hat{v}_i - \hat{v}_j = v_i - v_j + z_{vij}$ be the relative position and velocity of the robot *i* and node *j* in noisy environment, here $z_{vij} = z_{vi} - z_{vj}$ and $z_{pij} = z_{pi} - z_{qj}$. We have the error dynamic of the system as:

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2\\ \dot{\hat{x}}_2 &= \dot{v}_i - \dot{v}_j + \dot{z}_{vij}, \quad i,j = 1,2..,N.\end{aligned} \qquad (4)$$

However, in order to guarantee that the active neighboring robots while moving in a formation do not repel, the noise z_{pi} must satisfy $\left\|z_{pi}\right\| < r_{n1}$, here r_{n1} is a noise radius. This noise radius can be selected as depicted in Fig.1a, such that: $r_{n1} = \lambda/2$, here the positive factor $\lambda = d - r_r$ as shown in Fig.2. Moreover, the noise's amplitude must also guarantee that robots do not collide to each other during movement. Thus, we can choose another noise radius $r_{n2} = \lambda^*/2$, see Fig.1b, here $\lambda^* > 0$ is a region used to detect the collision among robots. Finally, in order to solve both above conditions the noise z_{pi} has to satisfy $\left\|z_{pi}\right\| < \min(r_{n1}, r_{n2})$.

Proposition:

Consider the active robot i with its dynamic model (1) and control input u_i^j given as (3) at the active node j of the desired formation in noisy environment. If the velocity of the node j is smaller than the maximum velocity of the robot i, and the node j-1 is also active, and the noise is bounded by $\left\|z_{pi}\right\| < \min\left(\lambda/2, \lambda^*/2\right)$, then the system (4) is stable at the equilibrium state $(\hat{p}_i \approx \hat{q}_j, \hat{v}_i \approx \hat{v}_j)$ for all i and j.

Proof of this proposition:

Consider the vector field $f_{2i}^j = -\hat{k}_{ip1}^j (\hat{p}_i - \hat{q}_j)^T = (P_i, Q_i, Z_i)^T$, here $P_i = -\hat{k}_{ip1}^j (\hat{x}_i - \hat{x}_j)$, $Q_i = -\hat{k}_{ip1}^j (\hat{y}_i - \hat{y}_j)$ and $Z_i = 0$. According to [19], we obtain:

$$rot(f_{2i}^j) = \left(\frac{\partial Z_i}{\partial y_i} - \frac{\partial Q_i}{\partial z_i},\ \frac{\partial P_i}{\partial z_i} - \frac{\partial Z_i}{\partial x_i},\ \frac{\partial Q_i}{\partial x_i} - \frac{\partial P_i}{\partial y_i}\right) = 0. \qquad (5)$$

Equation (5) shows that the vector field f_{2i}^j is irrotational. Consider the scale function as follows:

$$V_i^j = \frac{1}{2}\hat{k}_{ip1}^j (\hat{p}_i - \hat{q}_j)^T \left(\hat{p}_i - \hat{q}_j\right). \qquad (6)$$

Taking the negative gradient of the function V_i^j we obtain:

$$\begin{aligned}-\nabla V_i^j &= -\nabla\left(\frac{1}{2}\hat{k}_{ip1}^j (\hat{p}_i - \hat{q}_j)^T \left(\hat{p}_i - \hat{q}_j\right)\right)\\ &= -\nabla\left(\frac{1}{2}\hat{k}_{ip1}^j \left(\left(\hat{x}_i - \hat{x}_j\right)^2 + \left(\hat{y}_i - \hat{y}_j\right)^2\right)\right)\\ &= \begin{pmatrix}-\frac{1}{2}\hat{k}_{ip1}^j \frac{\partial}{\partial \hat{x}_i}\left(\left(\hat{x}_i - \hat{x}_j\right)^2 + \left(\hat{y}_i - \hat{y}_j\right)^2\right)\\ -\frac{1}{2}\hat{k}_{ip1}^j \frac{\partial}{\partial \hat{y}_i}\left(\left(\hat{x}_i - \hat{x}_j\right)^2 + \left(\hat{y}_i - \hat{y}_j\right)^2\right)\end{pmatrix}^T\\ &= \begin{pmatrix}-\hat{k}_{ip1}^j \left(\hat{x}_i - \hat{x}_j\right)\\ -\hat{k}_{ip1}^j \left(y_i - y_j\right)\end{pmatrix}^T = -\hat{k}_{ip1}^j (\hat{p}_i - \hat{q}_j) = f_{2i}^j.\end{aligned} \qquad (7)$$

So, (5) and (7) show that the vector field f_{2i}^j is a potential field, and its potential function is V_i^j.

Substitute u_i^j in (3) into (4) we obtain the error dynamic model of the system as follows:

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2\\ \dot{\hat{x}}_2 &= -\nabla V_i^j - \hat{k}_{iv}^j (\hat{v}_i - \hat{v}_j), \quad i,j = 1,2..,N.\end{aligned} \qquad (8)$$

To analyze the stability of model (8) at the equilibrium position $(\hat{p}_i - \hat{q}_j \approx 0,\ \hat{v}_i - \hat{v}_j \approx 0)$, the positive definite function is selected as follows:

$$V_z = V_i^j + \frac{1}{2}\hat{x}_2^T \hat{x}_2. \tag{9}$$

Consider the potential function (6) we have the relation $\left(\partial V_i^j / \partial \hat{p}_i\right)^T = \left(\partial V_i^j / \partial(\hat{p}_i - \hat{q}_j)\right)^T$. Taking the time derivative of (9) along the trajectory of the system (8), we obtain:

$$\dot{V}_z(t) = \nabla V_i^j \dot{\hat{x}}_1^T + \hat{x}_2^T \dot{\hat{x}}_2 = \hat{x}_2^T (\nabla V_i^j + \dot{\hat{x}}_2). \tag{10}$$

Substitute $\dot{\hat{x}}_2$ in (8) into (10) we obtain:

$$\dot{V}_z(t) = -\hat{k}_{iv}^j \hat{x}_2^T \hat{x}_2 \le 0. \tag{11}$$

So, equation (11) shows that the system (8) is stable with the control law (3). However, this stability is limited by the boundary of the noise. If $\|z_{pi}\| \ge \min\left(\lambda/2, \lambda^*/2\right)$, then the active neighboring robots can repel to each other or the robots can collide, so the stability is broken, see Fig.1.

B. *Target tracking, collision avoidance and obstacle avoidance control*

As discussed above, with the formation stability and maintenance following a desired formation in a free environment, robots must automatically escape obstacles in order to approach to the target without collisions. To perform these works, we use the control algorithms: collision avoidance control, obstacle control and target tracking control. According to [5, 6], these control algorithms are summarized as follows:

- *Target tracking control*

For the target tracking, the control law is given as:

$$u_l^t = f_l^t - k_{lv}^t(\hat{v}_l - \hat{v}_t) + \dot{\hat{v}}_t, \tag{12}$$

where, k_{lv}^t and $\dot{\hat{v}}_t$ are the positive factor and the acceleration of the target, respectively. The component $(\hat{v}_l - \hat{v}_t)$ is the relative velocity vector between the leader and the target. The potential field f_l^t from the target is used to drive the leader moving towards the target, and it is calculated as $f_l^t = \xi_l^t(\hat{p}_l - \hat{p}_t)/\|\hat{p}_l - \hat{p}_t\|$, here: ξ_l^t is a control element, and $(\hat{p}_l - \hat{p}_t)$ is the relative position between the leader and the target.

- *Collision avoidance control*

According to [5, 6], the collision avoidance controller, which is designed based on the combination of the repulsive potential field with the damping term, is given as follows:

$$\begin{aligned} u_i^k = &\sum_{\substack{k=1,\,k\neq i,\\ k\,\in N_i^k(t)}}^{N} \left(\left(\frac{1}{d_i^k} - \frac{1}{r_r} \right) \frac{k_{i1}^k}{(d_i^k)^2} - k_{i2}^k \left(d_i^k - r_r \right) \right) n_i^k \\ &- \sum_{\substack{k=1,\,k\neq i,\\ k\,\in N_i^k(t)}}^{N} k_{iv}^k (\hat{v}_i - \hat{v}_j). \end{aligned} \tag{13}$$

Where, r_r is the repulsive radius surrounding each robot, and d_i^k is the Euclidean distance between robot k and robot i. The positive factors $k_{i1}^k,\ k_{i2}^k,\ k_{iv}^k$ are used to control the fast interaction. The unit vector n_i^k from robot k to robot i is given as $n_i^k = (\hat{p}_i - \hat{q}_k)/\|\hat{p}_i - \hat{q}_k\|$. The set of the neighboring robots of the robot i at time t, is defined as follows $N_i^k(t) = \left\{\forall k : d_i^k = \|\hat{p}_i - \hat{q}_k\| \le r_r, k \in \{1,..N\}, k \ne i\right\}$.

The controller (13) shows that the neighboring robots are always driven to leave each other. In other words, this controller guarantees that there are no collisions among robots in the swarm.

- *Obstacle avoidance control*

The obstacle avoidance control algorithm for each member robot i ($i=1,2,..,N$) is designed as follows:

$$\begin{aligned} u_i^o = &\sum_{\substack{o=1,o\neq k\\ o\in N_i^o(t)}}^{M} \left(\left(\frac{1}{d_i^o} - \frac{1}{r^\beta} \right) \frac{k_{i1}^o}{(d_i^o)^2} - k_{i2}^o \left(d_i^o - r^\beta \right) \right) \mathrm{n}_i^o \\ &+ \sum_{\substack{o=1,o\neq k\\ o\in N_i^o(t)}}^{M} \left(w_i^o n_i^{or} - k_{iv}^o (\hat{v}_i - \hat{v}_o) \right). \end{aligned} \tag{14}$$

Where, the relative velocity vector ($v_i - v_o$) between the robot i and its neighboring obstacle o ($o=1,2,..,M$) is used as a damping term. The components $r^\beta > 0$ and $d_i^o = \|\hat{p}_i - \hat{p}_o\|$ are the obstacle detection range and the Euclidean distance between the robot i and the obstacle o, respectively. The set of the neighboring obstacles of the robot i at time t is also defined as $N_i^o(t) = \left\{\forall o : d_i^o \le r^\beta,\ o \in \{1,..M\}, o \ne k\right\}$. The positive factors $k_{i1}^o,\ k_{i2}^o,\ k_{iv}^o$ are used to control the fast obstacle avoidance, and the unit vector n_i^o is given as $n_i^o = (\hat{p}_i - \hat{p}_o)/\|\hat{p}_i - \hat{p}_o\|$. Additionally, in the rotational force, which is applied to drive robots to escape obstacles, w_i^o is a control factor and the unit vector n_i^{or} is chosen as $n_i^{or} = c_i^{or}\left((\hat{y}_i - \hat{y}_o)/d_i^o, -(\hat{x}_i - \hat{x}_o)/d_i^o\right)^T$.

IV. Simulation Results

In this section, we present the results of the simulations of the above proposed control algorithms. We test the stability of the formation under the influences of noise and the change of the formation angle.

A. *Preparation for simulation*

For these simulations, we assume that the initial velocities of the robots and target are set to zero. The initial positions of the robots are random. V-shape and circular shape formations are used to test the proposed control algorithms. According to [5, 6], the algorithm to generate the desired V-shape formation of the virtual nodes j ($j=1,2..,N$) is built as follows:

$$\begin{pmatrix} \hat{x}_j \\ \hat{y}_j \end{pmatrix} = \begin{cases} \begin{pmatrix} \hat{x}_l \\ \hat{y}_l \end{pmatrix} - \xi_1 \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} d_d \cos\delta_d \\ d_d \sin\delta_d \end{pmatrix}, \\ \text{if } j \le N/2 + 1;\ j = 1,2..N, \\ \begin{pmatrix} \hat{x}_l \\ \hat{y}_l \end{pmatrix} + \xi_2 \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} -d_d \cos\delta_d \\ d_d \sin\delta_d \end{pmatrix}, \\ \text{otherwise}, \end{cases} \tag{15}$$

and the algorithm to generate the desired circular shape formation of the virtual nodes j ($j=1,2..,N$) is also designed as:

$$\begin{pmatrix}\hat{x}_j\\ \hat{y}_j\end{pmatrix}=\begin{pmatrix}\hat{x}_t\\ \hat{y}_t\end{pmatrix}+d_l^t\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}cos(2\zeta\pi/N)\\ sin(2\zeta\pi/N)\end{pmatrix}, \qquad (16)$$

$$j=1,2..N; \zeta=j-1.$$

Here: $\theta=\angle((\hat{p}_t-\hat{p}_l),\ ox)$, $\xi_1=j-1,\ \xi_2=j-1-floor(N/2)$, $\hat{q}_j=(\hat{x}_j,\hat{y}_j)^T$, $\hat{p}_t=(\hat{x}_t,\hat{y}_t)^T$ and $\hat{p}_j=\left(\hat{x}_j,\hat{y}_j\right)^T$. d_d=constant and $\varphi_d=\delta_d$ are the desired distance between the neighboring virtual nodes and the desired formation angle, respectively. In these case studies, we design the control element ξ_l^t in (12) as follows:

$$\xi_l^t=\begin{cases}\left(\dfrac{1}{d_l^t}-\dfrac{1}{r^\tau}\right)\dfrac{k_{l2}^t}{(d_l^t)^2}-\dfrac{k_{ll}^t(d_l^t-r^\tau)}{(r^t-r^\tau)}, & \text{if } d_l^t\le r^t\\ -k_{ll}^t, & \text{otherwise,}\end{cases} \qquad (17)$$

where, k_{l2}^t, r^t and r^τ are the positive constant, the target approaching radius, and the desired radius of the circular formation $r_{min}^t<r^\tau<r^t$, respectively. $d_l^t=\|\hat{p}_l-\hat{p}_t\|$ is the distance between the leader and the target. $k_{ll}^t=k_{lld}^t+\varepsilon_3\|v_t\|$, here k_{lld}^t, ε_3 are the positive factors. Equation (17) shows that a formation is driven by the leader will track a moving target when $d_l^t>r^t$. In contrast, it will approach and encircle the target. The noises used in this simulation are Gaussian function with zero mean, variance of 1 and standard deviation of 1, see Fig.3. The formation angle $\varphi(t)$ is utilized as follows: $\varphi(t)=2\pi/3+1.6sin\left(0.2t\right)$, see Fig.4. The target moves on a sine wave trajectory as follows: $p_t=(0.9t+640,\ 160\sin(0.01t)+250)^T$. The general parameters of the simulations are listed in table I.

TABLE I: PARAMETER VALUES

Parameter	Definition	Value
d_d	Desired distance between robots	60
λ	Positive constant	20
N	Number of robots	9
φ_d	Desired formation angle	2π/3
r^τ	Target approach radius	60
r_a	Radius around each active node	25
k_{l1}^t, k_{l2}^t	Factors for approach to target	9, 0.6
k_{i1}^j, k_{i2}^j, k_{i3}^j	Positive constants	3, 4, 9
k_{iv}^j, k_{iv}^t, k_{iv}^k,	Damping factors	1.4

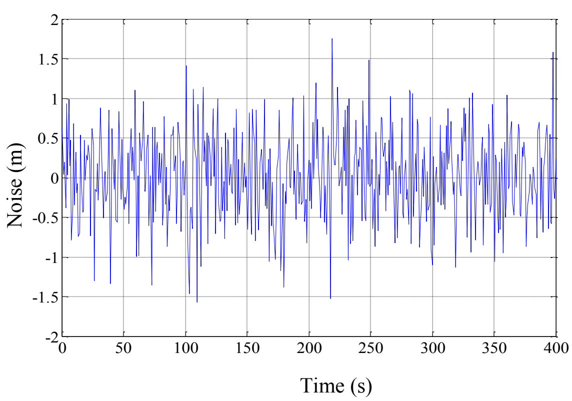

Fig.3. Noise effects on the system.

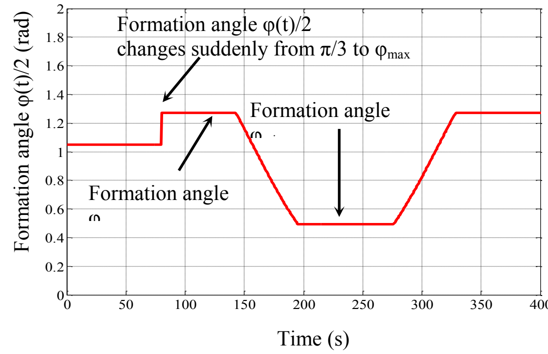

Fig.4. Formation angle φ(t)/2 while tracking a moving target.

B. Test the stability of the swarm in noisy environment without obstacles

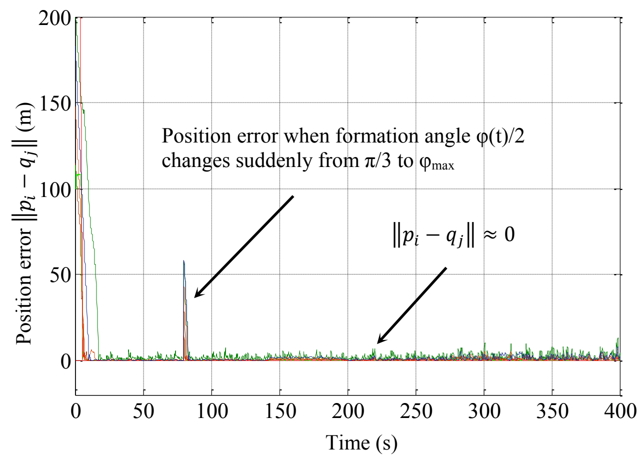

Fig.5. Position errors $\|p_i-q_j\|$ under the effect of the noise.

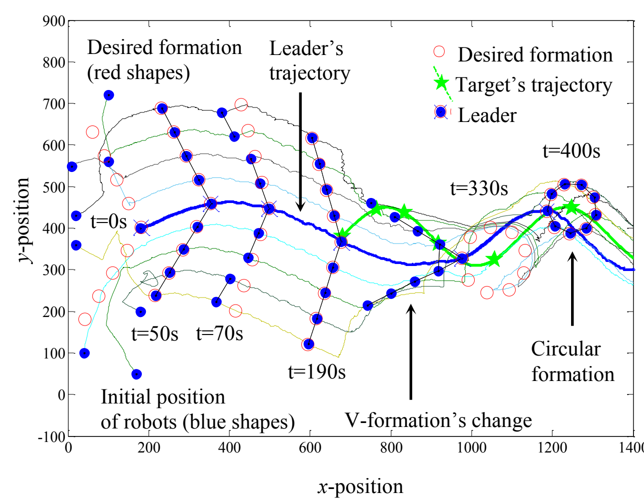

Fig.6. The influences of noises and the different formation angles on the swarm's trajectory while tracking a moving target.

The results in Fig.5 show that the robot i is always closed to the active node j in the desired formation, and its formation was maintained following the desired formations (V-shape and circular formation) although there are the effects of the noisy environment and the changes of the formation angle $\varphi(t)/2$. The position error between each robot i and the active node j, at which this robot i was occupying, is small, see Fig.5 and Fig.6. The simulation results in Fig.6 also shows that from the random initial positions, the free robots have quickly found their desired position on the desired V-formation. Then, they tracked a moving target in a stable V-formation. At time *t=70s*, un-der the influence of the sudden change of the formation angle from $2\pi/3$ to $\varphi_{max}=(\pi-0.6)$ the stability of the formation was broken, but it was quickly redesigned to continue to track the moving target. In contrast, when the formation angle $\varphi(t)$ changed slowly the formation of robots was always main-tained following the desired V-formation with the small position errors, see Fig.5, Fig.6. Moreover, the simulation results also show that the noise had influences to the position error on the robot formation, but this influence only caused small formation changes as shown in Fig.5.

C. *Test the stability of the swarm in noisy environment with obstacles*

The simulation results in Fig.7 also show that robots can easily escape obstacles, and always converge to the designed virtual nodes in the desired formations although there are effects of the noisy environment.

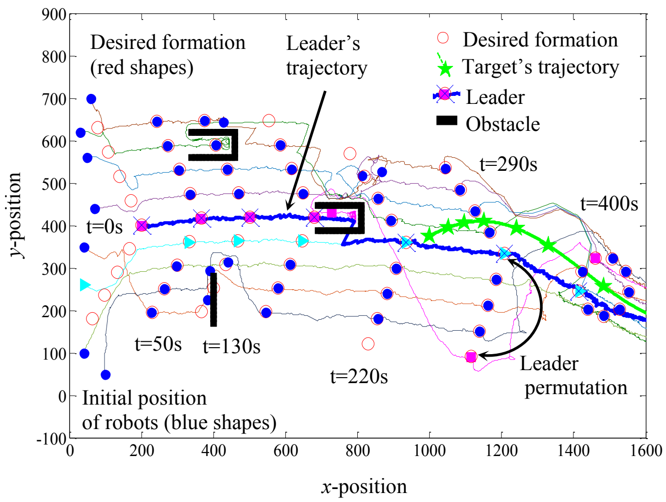

Fig.7. Path planning for a swarm following the desired formations while tracking a moving target in a noisy environment.

V. CONCLUSION

This paper presents a novel approach to formation cont-rol of autonomous robots following the desired formations to track a moving target in a dynamic and noisy environment. The robot team is able to form predefined formations such as V-shape or circular shape while tracking a moving target. The stability and convergence analysis of the proposed for-mation control is given. The rotational force field combining with the repulsive force can drive the robot to quickly escape from the obstacles, more importantly is to avoid the local minimum problems when the sum of the attractive and rep-ulsive forces of the potential field is equal to zero in the case of concave obstacle shapes. The development and applica-tion of this proposed approach for formation control of the flight robots in 3D space, such as formation of the unman-ned aerial vehicles will be our future research.

REFERENCES

[1] Shi and Yiwen Zhao, "An efficient path planning algorithm for mobile robot using improved potential field", *Proc. of the 2009 IEEE Intl. Conf. on Robotics and Biomimetics, pp. 1704-1708, December 2009.*

[2] Feilong Li, Yan Tan, Yujun Wang and Gengyu Ge, "Mobile robots path planning based on evolutionary artificial potential fields approach", *Proc. of the the 2th Intl. Conf. on Computer Science and Electronics Engineering, pp. 1314-1317, 2013.*

[3] Frank E. Schneider and Dennis Wildermuth, " A potential field based approach to multi robot formation navigation", *Proc. of the 2003 IEEE Intl. Conf. on Robotics, Intelligent System and Signal Processing, pp. 680-685,October 2003.*

[4] Anh Duc Dang and Joachim Horn, "Intelligent swarm-finding in formation control of multi-robots to track a moving target", *Intl. Journal of Computer, Information Science and Engineering, Vol.8, No. 4, pp.12-18,2014.*

[5] Anh Duc Dang and Joachim Horn, "Formation control of autonomous robots to track a moving target in unknown environment", *IEEE Intl. Conf. on Robotics and Biomimetics, pages 896-901, December, 2014.*

[6] Anh Duc Dang and Joachim Horn, "Formation control of autonomous robots following desired formation during tracking a moving target", *Proceedings of the 2nd IEEE International Conference on Cybernetics, Gdynia, Poland, pages 160-165, June 24-26,* 2015.

[7] Kishorekumar H Kowdiki, Ranjit Kumar Barai and Samar Bhattacharya, "Leader-follower formation control using artificial potential functions: A kinematic approach", *IEEE Intl. Conf. on Advances in Engineering, Science and Management, pp. 500-505,March 2012.*

[8] Chang Boon Low, "A dynamic virtual structure formation control for fixed-wing UAVs", *IEEE Intl. Conf. on control and Automation, pp. 627-632,December, 2011.*

[9] Huizhen Yang and Fumin Zhang,"Geometric formation control for autonomous underwater vehicles", *IEEE Intl. Conf. on Robotics and Automation, pp. 4288-4293, May, 2010.*

[10] S. P. Hou, C. C. Cheahand J. J. E. Slotine, "Dynamic Region Following Formation Control for a Swarm of Robots", *IEEE Intl. Conf. on Robotics and Automation, pp. 1929-1934, May, 2009.*

[11] Aveek Das, Rafael Fierro, Vijay Kumar, Jim Ostrowski, John Spletzer and Camillo Taylor, "A framework for vision based formation control", *IEEE Transactions on Robotics and Automation,Vol. XX, No. Y, Month, pages 1-13, 2001.*

[12] John R. Spletzer and Rafael Fierro.*"Optimal positioning strategies for shape changes in robot teams" IEEE Intl. Conf. on Robotics and Automation, pp. 742-747, April, 2005.*

[13] Tolga Eren, "Formation shape control based on bearing rigidity", *Intl Journal of control, Vol. 85, No. 9, pages 1361–1379, September, 2012.*

[14] H. M. La and W. Sheng, "Dynamic targets tracking and observing in a mobile sensor network", *the Elsevier journal on Robotics and Autonomous Systems*, pp. 996–1009, *Vol. 60, Issue 7, July 2012.*

[15] R. Olfati-Saber, "Distributed tracking for mobile sensor networks with information driven mobility". *Proceedings of the 2007 American Control Conference, pages 4606–4612, 2007.*

[16] H. M. La and W. Sheng, "Distributed sensor fusion for scalar field mapping using mobile sensor networks", *IEEE Transactions on Cybernetics, pp. 766-778, Vol. 43, No. 2, April, 2013.*

[17] H. M. La, R. Lim and W. Sheng, "Multi-robot cooperative learning for predator avoidance", *IEEE Transactions on Control Systems Technology, Vol. 23, No. 1, pp. 52-63, January, 2015.*

[18] H. M. La, W. Sheng and J. Chen "Cooperative and Active Sensing in Mobile Sensor Networks for Scalar Field Mapping", *IEEE Transactions on System, Man, and Cybernetics: System, Vol. 45, No. 1, pp. 1-12, January, 2015.*

[19] Lial and Hungerford, "Mathematics with applications", *8th edition, Addison Wesley, 2003.*